A Comparison of Neural Network Architectures in Reinforcement Learning in the Game of Othello

by

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Declaration

This thesis contains no material which has been accepted for the award of any other degree or diploma in any tertiary institution, and to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference is made in the text of the thesis.

Signed

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Abstract

Over the past two decades, Reinforcement Learning has emerged as a promising Machine Learning technique that is capable of solving complex dynamic problems. The benefit of this technique lies in the fact that the agent learns from its experience rather than being told directly. For problems with large state-spaces, Reinforcement Learning algorithms are combined with function approximation techniques, such as neural networks. The architecture of the neural networks plays a significant role in the agent's learning. Past research has demonstrated that networks with a constructive architecture outperform those with a fixed architecture on some benchmark problems.

This study compares the performance of these two architectures in Othello – a complex deterministic board game. Three networks are used in the comparison: two with constructive architecture – Cascade and Resource Allocating Network, and one with fixed architecture - Multilayer Perceptron. Investigation is also made with respect to input representation, number of hidden nodes and other parameters used by the networks. Training is performed with both on-policy (Sarsa) and off-policy (Q-Learning) algorithms.

Results show that agents were able to learn the positional strategy (novice strategy in Othello) and could beat each of the three built-in opponents. Agents trained with Multilayer Perceptron perform better, but converge slower than those trained with Cascade.

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Chapter 1 Introduction

Computers are ideally suited for calculation-intensive and repetitive tasks. They can plough through masses of data without getting tired or making any mistakes. But is that all they can do or can we make them think and make decisions like we humans can? It was this question that has always fascinated researchers and started the field of Artificial Intelligence. Following in the steps of those before us we too attempt to find the answer.

Teaching a computer to think should be similar to teaching a child or an animal. A common method of training animals is to reinforce their actions with rewards and punishments. For example to teach the animal a particular action, trainers reward the animal with food when it performs that action. Similarly, an undesirable action can be eliminated by appropriately punishing the animal. Reinforcement Learning is an Artificial Intelligence technique that applies the above methods to computing. The concept has not changed – the computer agent learns from its experience via a system of rewards and punishments that are represented by integer values. The agent receives input describing the current state of the environment and must select actions that maximize its long-term reward.

Early work in Reinforcement Learning has mostly concentrated on problems with small state-spaces, where each state can be stored in memory. These problems are great for validating new algorithms, but are not complex enough for detailed analysis. As we increase the complexity of a problem, storing every state becomes impractical. For such problems, Reinforcement Learning algorithms must be combined with function approximation techniques, such as neural networks. Neural networks come in many shapes and sizes, but most work has involved fixed architecture networks. These networks, otherwise known as MLPs, have a predetermined number of units and this number does not change during training. MLPs have been successful in a number of problem domains, especially in Backgammon, where an agent trained via self-play has been able to match the performance of world-class human players (Tesauro 1992, 1994, 1995). In other domains however, fixed-architecture networks

perform poorly, as demonstrated by their failure in the two spirals problem (Lang & Witbrock 1988).

In 1990 Fahlman and Lebiere introduced a new type of neural network, called the constructive neural network. Constructive neural networks begin with no hidden nodes and add new hidden nodes one at a time during training. Due to the nature of their training algorithm, these networks are able to recognize complex patterns, while forming compact solutions at the same time. They have outperformed MLPs in the two spirals problem (Fahlman & Lebiere 1990) and on some benchmark Reinforcement Learning problems (Vamplew & Ollington 2005b).

There has been no research that compares these two network architectures in a more complex environment and hence that is the aim of this project. Due to its relatively high complexity (~10^{28} states), the game of Othello was chosen as a testing environment. To test the true learning ability of the networks, no game-tree search was used and the provided input contained only the bare essentials describing the state of the game.

Chapter 2 Game-playing with Artificial Intelligence

2.1 History

Since the advent of personal computers, computer games have been an ideal domain for exploring the capabilities of Artificial Intelligence (AI). Unlike real-world problems, the rules of games are fixed and well defined, the search space is constrained and the variables are discrete. Games research acts a stepping stone towards solving more complex real-world problems (Schaeffer 2001).

Research into computer games was begun in 1950 by Shannon, who attempted to describe a program that could play Chess. In his phenomenal work, Shannon laid down the foundations for all future game-playing programs (Shannon 1950). A decade later, Arthur Samuel introduced his famous Checkers-playing program (Samuel 1959, 1967). One of the earliest goals of AI researchers was to build a program that could challenge human world champions in the game of Chess. This task proved to be more difficult than originally envisioned. It took 50 years, as well as great leaps in technology and algorithmic knowledge before the task could be completed. In an exhibition match in 1997 a machine called *Deep Blue* became the first program to defeat the reigning Chess grandmaster Garry Kasparov.

A great deal of success has been achieved in numerous games. Some games have been solved, meaning that there exists an optimal strategy that allows a player to force a draw or better. Solved games include Connect Four (Allis 1988), Dakon (Donkers et al. 2000), Go Moku (Allis et al. 1995), Kalah (Irving et al. 2000), Nine-Men's Morris (Gasser 1996), Qubic (Allis 1994) and Tic-Tac-Toe with its varieties (Uiterwijk & van den Herik 2000). For other games there exist programs that can equal or even exceed the abilities of the best human players. These games include Backgammon, Checkers, Chess, Draughts, Othello and Scrabble (Schaeffer 2000; van den Herik et al. 2002). However the pinnacle for AI research remains the ancient Chinese game Go. Computer Go is still in its infancy with the best computer programs only playing at the level of an amateur human. Go has a massive search space of 10^{172} moves which make it infeasible for the standard game tree search approach.

2.2 Introduction

The most difficult aspect of playing a game is selecting the best action of play for a given situation. When selecting a move, the player must consider all available moves and choose a move which is likely to lead to a favourable situation for that player. The quality of a move can be evaluated in a number of ways, with some measures being more accurate than others. For simple games like Tic-Tac-Toe it is possible to find the perfect evaluation function; a function that provides perfect information about every move and guarantees a draw or better. However for most games there exists no such function and hence game-playing agents must rely on approximations to the perfect function. Artificial Intelligence approaches to game-playing divide into two categories. The first category aims to improve the evaluation function to bring it closer to the perfect function. The second category accepts the fact that the evaluation function is not perfect and attempts to improve the agent's playing abilities by other means, such as searching a game tree.

2.3 Game Tree Search

Many approaches to game playing involve the process of searching a game tree. Each node in a game tree represents a particular state in the game, such as the board configuration. Nodes are linked by branches that signify transitions or moves from one game state to another. The standard approach to game tree searching is the minimax approach. Although the minimax approach is accurate it can be inefficient since it has to consider all the states in a game tree. Alpha-beta and heuristic pruning are techniques that aim to reduce the number of nodes that need to be considered by minimax. Other improvements to minimax include progressive deepening.

2.3.1 Minimax

The minimax approach was introduced by Claude Shannon who was well ahead of his time (Shannon 1950). Suppose we have a function f: GameState → Value. f takes the

current state of the game and returns a value reflecting the likelihood of the first player winning from the given state of the game. If the returned value is positive then the first player has a better chance of winning than his opponent. If the returned value is negative then the first player is more likely to loose as his opponent is in a winning position. Finally if the value is close to zero then both players are equally likely to win from here.

In a two-player game, the first player always attempts to maximize the output of f, while the second player tries to minimize this output. These ideas lead directly to the minimax approach to game-playing. In minimax, the first player is called the *maximizing* player, while his opponent is the *minimizing* player. The goal of the maximizing player is to follow a path that leads to a state s where f(s) is maximal. The maximizing player assumes that his opponent is doing the opposite by forcing play to lead to small state values (Winston 1984). Thus in a game tree, the score at each node is either the minimum or the maximum of its sibling nodes; hence the name minimax.

To illustrate how the minimax procedure is used in practice, consider the game tree in Figure 2.1:

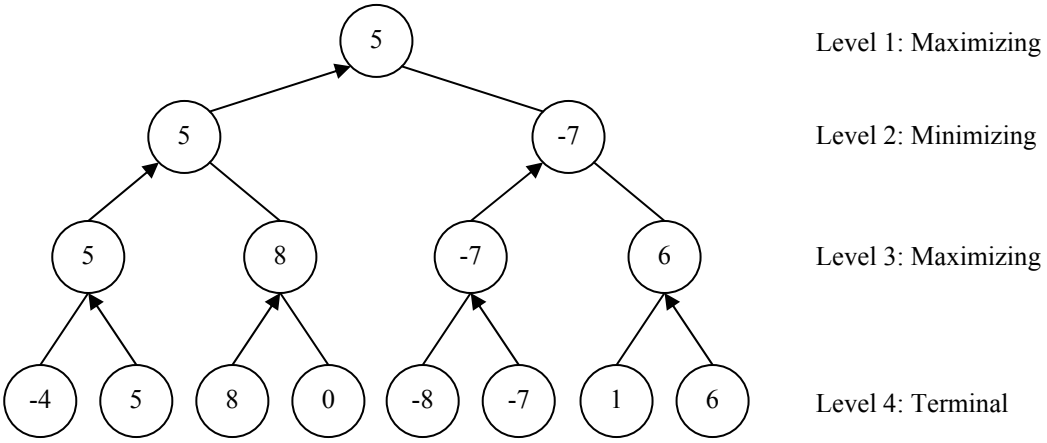


Figure 2.1 Demonstration of the minimax procedure.

The first level of the tree contains just the root node, which is the current state of the game as evaluated by the maximizing player. The next level contains the states as viewed by the minimizing player. In a similar fashion, levels 3 and 4 alternate between the maximizer and minimizer respectively.

Potentially the maximizer can get a score as high as 8. However, the maximizer knows that the minimizer will not permit that, because in level 2 the minimizer will choose a state with value 5 over the state with value 8. In general, the decision of the maximizer must take into account the decision of the minimizer at the next level down. At the same time, the decision of the minimizer at the next level is dependant on the decision of the maximizer at a level below. This process continues until the terminal nodes of the bottom level are reached, where we have a static evaluation of the game states.

In this example, the static values at level 4 provide information for the maximizer at level 3. The maximizer chooses the maximum of each pair of numbers. At level 2, the minimizer chooses the minimum from each pair of level 3. Finally at the top level, the maximizer chooses 5 over -7 to get the final evaluation of the current state.

2.3.2 Alpha-beta pruning

The idea of alpha-beta pruning is to reduce the number of nodes that need to be considered in the minimax search. This reduction is possible when sufficient information has been gathered for us to make a conclusion about the value of a parent node, thus eliminating the need to consider other children. The method uses two bounds α and β, which are passed down as the tree is traversed. At any node, α and β represent the smallest and largest node values respectively that can affect the minimax value above that node. Thus, α and β are often referred to as the *search window* (Brockington 1998). To demonstrate alpha-beta pruning we use the game tree from Figure 2.1. This time however, nodes that do not need to be considered are left blank (Figure 2.2).

Once the maximizer sees 8 at level 4, he knows that the value of b will be 8 or more. Since the minimizer at level 2 chooses the minimum between 5 and b, he knows that b cannot be chosen and selects 5 instead. Thus there is no point for the maximizer to look at the right node of b.

Once the minimizer has seen -7 at level 3, he knows that the value of a will be -7 or

less. Since the maximizer at level 1 chooses the maximum between 5 and a, he knows that a cannot be chosen and selects 5 instead. Thus there is no point for the minimizer to look at the right sub-tree of a.

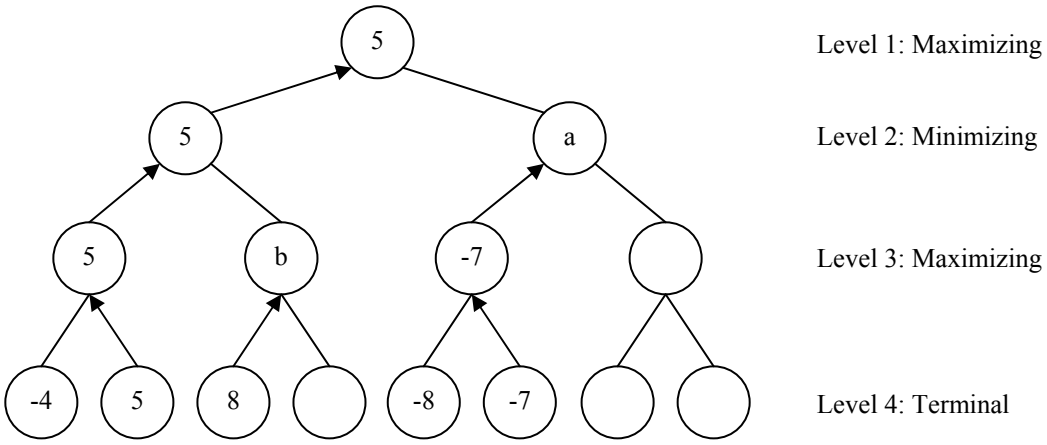


Figure 2.2 Demonstration of alpha-beta pruning applied to the minimax procedure.

It must be understood that alpha-beta pruning has its own limitations and may not always be beneficial. For example, for some trees, the branches can be ordered in such a way that alpha-beta pruning does nothing (Winston 1984).

Let s be the number of static evaluations needed to discover the best move, b be the branching factor and d the depth of the tree. For a simple minimax search we have $s = b^d$. If we apply alpha-beta pruning to minimax then in the best-case scenario the above equation becomes the following:

$$s = \begin{cases} 2b^{d/2} - 1 & \text{if } d \text{ is even} \\ b^{(d+1)/2} + b^{(d-1)/2} - 1 & \text{if } d \text{ is odd} \end{cases}$$

Equation 2.1 Complexity of the alpha-beta algorithm.

Equation 2.1 tells us that applying alpha-beta pruning reduces the number of static evaluations to approximately $2b^{d/2}$. Although this is a clear improvement on the simple minimax search, the exponential growth of the tree has not been eliminated.

2.3.3 Heuristic Pruning

Heuristic methods, like alpha-beta pruning, attempt to reduce the number of static evaluations needed for a game tree. However, unlike alpha-beta pruning, heuristic methods do not guarantee to retain the optimal result of a minimax search. Hence these methods should be used with great caution. It seems obvious that we can reduce the number of static evaluations by concentrating only on the best moves. Once the best moves are known we can alter the branching factor so that these moves are explored further than other moves:

$$BF(x) = BF(parentOf(x)) - rankOfNode(x)$$

Equation 2.2 Proposed branching factor for heuristic pruning.

In Equation 2.2 *BF(x)* is the branching factor for a node *x*, *BF(parentOf(x))* is the branching factor for the parent node of *x* and *rankOfNode(x)* is the rank in plausibility of the node *x* among its siblings. For example, if a node is one of seven children and ranks third most plausible amongst those seven, then it should itself have 7 – 3 = 4 children. This method not only concentrates on the best moves, but also avoids bad moves, that could potentially lead to disastrous future situations (Winston 1984).

2.3.4 Progressive Deepening

In a tournament environment there are usually bounds set on the time limit allowed for each move. In those situations it may be impractical to search a game tree to a fixed depth, since the search may take too long and no final decision will be made. Progressive deepening is a commonly used technique that combats this problem. As the name suggests, the technique searches the game tree one level at a time, progressively deepening further into the tree. At each new level the best currently available move is recorded. This way, there is always a move available before the time limit is reached (Winston 1984).

2.4 Function improvement

As an alternative to search, game-playing systems can be improved by increasing the accuracy of the board evaluation function. In principle, an ideal board evaluation function guarantees to provide a correct evaluation for every possible board state, thus eliminating the need for search. In reality however, such ideal functions can only be generated for games with small game trees. For all other situations we must rely on function improvement. There are two main methods by which functions can be optimised: through natural selection - genetic algorithms and through experience – Reinforcement Learning.

2.4.1 Genetic Algorithms

Genetic algorithms are well-suited for improving the evaluation function because they are unlikely to get 'trapped' in bad local maxima. This property makes them perfectly suited for complex domains. Furthermore, they do not depend on gradient information and perform well where this information is not readily available.

Genetic algorithms have been successfully applied to Checkers (Chellapilla & Fogel 1999, 2001), Othello (Alliot & Durand 1996; Moriarty & Miikkulainen 1995) and recently to Chess (Fogel et al. 2004). Genetic algorithms operate by evolving a population of candidate solutions, whose aim is to maximize a given function. The output of this function is used as a direct measure of the fitness of each candidate solution. Resembling natural selection, solutions with the highest fitness are chosen. These chosen solutions are then recombined and mutated to produce the next generation of candidate solutions. The new candidates compete based on their fitness with the old candidates for a place in the next generation. This process is repeated until a sufficiently good candidate is found or until there is no more improvement between successive generations (Eiben & Smith 2003).

2.4.2 Reinforcement Learning

In Reinforcement Learning (RL) the agent learns from its experience. The agent is rewarded for winning and is punished for losing games. Based on this system, the agent learns which moves lead to a winning situation and which moves should be avoided. RL algorithms do not require a complete model of the game, but only its rules and final outcomes. A popular RL algorithm is Temporal Difference (TD) learning. In TD, the estimated value of the current board is updated based on the immediate reward and the estimated value of the subsequent board. In the case of a deterministic game like Othello, RL algorithms can be extended to learn the values of subsequent states (afterstates), instead of the usual state-action values. If n states lead to the same afterstate then by visiting just one of those states, the agent can assign correct values to all n states (Sutton & Barto 1998).

RL algorithms have been extensively applied to games. The most successful application was in Backgammon (Tesauro 1992, 1994, 1995), where a program (TD-Gammon) trained via self-play was able to match top human players. Other applications include: Chess (Beal & Smith 2000; Dazeley 2001; Thrun 1995), Connect Four (Ghory 2004), Go (Schraudolph et al. 2000), Othello (Tournavitis 2003; Walker et al. 1994) and Shogi (Beal & Smith 1999).

Chapter 3 Reinforcement Learning

3.1 Introduction

Reinforcement Learning (RL) is a Machine Learning technique, which has become very popular in recent times. The technique has been applied to a variety of artificial domains, such as game playing, as well as real-world problems. In principle, a Reinforcement Learning agent learns from its experience by interacting with the environment. The agent is not told how to behave and is allowed to explore the environment freely. However once it has taken its actions, the agent is rewarded if its actions were good and punished if they were bad. This system of rewards and punishments teaches the agent which actions to take in the future, and guides it towards a better outcome.

Usually in RL tasks the agent is not rewarded after each and every action. This creates what is known as the "temporal credit assignment problem" - when the agent is eventually awarded, how does it know which actions were most responsible for that award? Temporal Difference (TD) learning is a RL algorithm that deals with this problem. In TD learning, the agent learns which states of the environment correlate highly with rewards, and then uses this knowledge to decide which actions to take.

Reinforcement Learning should not be confused with supervised learning. Unlike in supervised learning, a Reinforcement Learning agent is not provided with input/output pairs. Instead, the agent is only given the immediate reward and the next state. The agent must rely on its experience of possible states, actions, transitions and rewards to be able to act optimally (Kaelbling et al. 1996).

3.2 History

In the early stages of Artificial Intelligence several researchers began to explore trial-and-error learning. In 1954, Minsky discussed computational models of

Reinforcement Learning. He described the construction of an analog machine composed of components he called SNARCs (Stochastic Neural Analog Reinforcement Calculators). Also in 1954, Farley and Clark described a neural-network learning machine designed to learn by trial and error. In 1960s the terms "reinforcement" and "reinforcement learning" were used for the first time in literature. In 1961 Minsky was the first to discuss how to distribute credit among the many actions that led to a particular goal (credit assignment problem). It is this problem that the methods of Reinforcement Learning have been trying to solve. In 1968, Michie and Chambers introduced a Reinforcement Learning controller called BOXES, which they applied to the pole-balancing task. The pole-balancing task was one of the earliest tasks involving incomplete knowledge and it soon became the testing ground for many future RL algorithms. In 1972, Klopf became the first to make a distinction between supervised and Reinforcement Learning. He argued that supervised methods prevented the agent from developing adaptive behaviours, which enable it to control the environment toward desired goals and away from undesired outcomes. Klopf's ideas formed the foundation of Reinforcement Learning as we know it today.

3.3 Environment

The agent cannot exist on its own. It must interact with its surroundings by performing actions and receiving rewards if those actions were deemed appropriate. In Reinforcement Learning the agent's surroundings are called the environment.

The agent interacts with the environment at each discrete time step t = 0, 1, 2, 3, … At each time step the agent receives the environment's representation of the state s_t from a set of possible states S. Based on state s_t, the agent selects an appropriate action a_t from a set of available actions for that state $A(s_t)$. As the consequence of the action a_t, the agent receives a reward r_{t+1} and finds itself at a new state s_{t+1} one time step later (Sutton & Barto 1998).

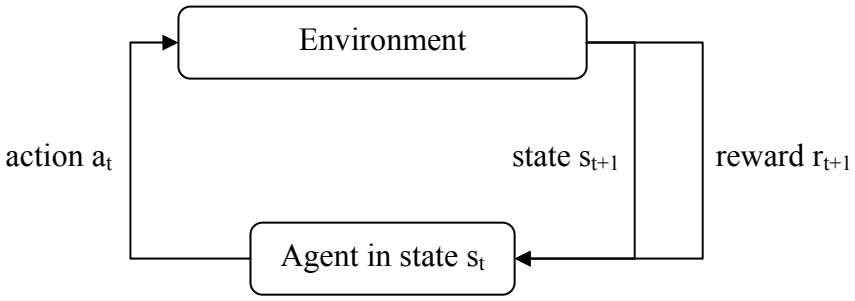


Figure 3.1 The agent-environment interaction in Reinforcement Learning.

3.4 Reward

At each time step the agent receives a reward $r_t \in \Re$. This reward is a numerical value that can act as either encouragement or punishment for the agent's actions. As a general rule, the reward is positive if the prior action was desirable and negative if that action was undesirable. The agent's goal is to maximize the total reward received (Sutton & Barto 1998). For example, when teaching a robot to escape a maze, the reward is set at 0 while it is still inside the maze. Once the robot escapes it is given +1. To encourage the robot to escape the maze as quickly as possible a common approach is to set the reward to -1 for every time step that passes prior to escape (Koenig & Simmons 1996). Sometimes it is useful to reward the agent for achieving partial goals, rather than the final goal. For example, we can reward a Chess-playing agent every time it takes a piece and penalize it when it loses a piece. This way the agent will learn to have more pieces. However this reward scheme can be detrimental, since having more pieces does not necessarily lead to a win. We must be careful when rewarding the agent for partial goals, because it might lose sight of the bigger picture of winning the game (Sutton & Barto 1998).

3.5 Return

The total accumulated reward is called the *return* and is denoted by R_t. In its simplest form, R_t is the sum of all rewards starting at time t + 1 and ending at the final step T. Note that T is finite for episodic tasks and infinite for continuing tasks, i.e. tasks

without a final state. This simplistic version of R_t works on the assumption that all rewards are equally important no matter when they occur. However this may not always be the case, as immediate rewards tend to be more important than future rewards. A common modification to R_t is the introduction of a discount rate γ, where $0 \leq \gamma \leq 1$ (Sutton & Barto 1998):

$$R_t = r_{t+1} + \gamma\ r_{t+2} + \gamma^2 r_{t+3} + \ldots = \sum_{k=0}^{T} \gamma^k r_{t+k+1}$$

Equation 3.1 Expected return.

The discount rate determines the value of future rewards: a reward received k time steps in the future is worth γ^{k-1} less than if the same reward was received now. As long as γ is less than 1 and the reward sequence r_k is bounded then R_t is finite. If $\gamma = 0$ then the agent is only concerned about maximizing the immediate reward. As γ approaches 1 the agent becomes more farsighted and concerned about future rewards. The final goal of the agent is to maximize its return (Sutton & Barto 1998).

3.6 Policy

The agent's policy defines how the agent behaves for a given situation. In the gaming context, the policy is the playing strategy that the player utilizes. The policy π is a mapping from each state $s \in S$ and action $a \in A(s)$ to the probability $\pi(s,a)$ of selecting action a when in state s (Sutton & Barto 1998).

The policy is constantly evolving as the agent becomes more familiar with its environment. At the start of learning, when the policy has not been fully established, the agent will select actions almost randomly. As the agent becomes more experienced it will be able to recognize some actions as being "better" than others and hence it will be more likely select those "better" actions. Generally, action a_1 is considered "better" than action a_2 if a_1 leads to a greater return. There is always at least one policy that is better than or equal to all other policies. This policy is called the optimal policy and is denoted by π^* (Sutton & Barto 1998).

3.7 Exploration and exploitation

An important factor that needs to be considered by a Reinforcement Learning agent is the balance between exploration and exploitation. In order to obtain rewards the agent must explore the environment to find states that yield the rewards. At the same time, the agent would much rather try states that have been rewarding in the past, i.e. exploit previous knowledge. Exploration and exploitation must go hand in hand for the agent to be successful. However, the agent must wisely balance their relative amounts. If there is not enough exploration then there will be a potential goldmine of unexplored states. On the other hand, insufficient exploitation can lead to a lack of received reward.

When choosing an action, the agent should not simply select the best available action. Occasionally it should select a lower ranked exploratory action as it may lead to the discovery of a better strategy. The following section describes two common approaches for selecting exploratory actions: ε-greedy and softmax selection.

3.7.1 ε-greedy Selection

In ε-greedy action selection, ε is the probability that the action will be chosen at random. If the action is not chosen at random then the best available action is selected. At the start of learning ε is set to a rather high value, such as 0.2, to accommodate exploration. As the agent becomes more familiar with the environment, ε is gradually decreased. As ε approaches 0 the agent becomes more concerned with exploitation rather than exploration.

$$\varepsilon - \text{greedy}(\varepsilon, \kappa, A) = \begin{cases} \text{any } a \in A, & \text{if } \kappa \le \varepsilon \\ a \in A \text{ such that } val(a) \ge val(b)\ \forall b, & \text{otherwise} \end{cases}$$

Equation 3.2 ε-greedy action selection algorithm.

In Equation 3.2 ε is the greediness factor, κ is some random number, and A is the set of available actions. ε and κ are between 0 and 1 inclusive.

3.7.2 Softmax Selection

Although ε-greedy is easy to implement it has some major weaknesses. One disadvantage of ε-greedy is that it can choose an action randomly, even if that action is completely useless. Choosing the best action as a random choice is not wise either, as you miss out on the opportunity of exploring the nearly-best actions. It is those nearly-best actions that may lead to a better policy. Softmax is an action-selection algorithm that attempts to rectify these problems. Softmax takes into account the relative values of each action by assuming that they have a Boltzmann distribution. An action whose value is high has a high probability of being chosen; similarly an action whose value is low has a low probability of being chosen; finally if two actions have a similar value then they are both as likely to be chosen. Equation 3.3 gives the probability of an action being chosen using softmax selection:

$$probability(\tau, a, A) = \frac{e^{val(a)/\tau}}{\sum_{i \in A} e^{val(i)/\tau}}$$

Equation 3.3 Probability of choosing an action a using softmax selection algorithm.

In Equation 3.3 τ is a non-negative value called *temperature*. The temperature allows us to control the balance between exploration and exploitation (Wyatt 1997). High temperatures cause probabilities to be nearly equal, hence promoting exploration. As τ approaches 0 the probabilities become unevenly distributed making softmax the same as greedy action selection. In practice the temperature is slowly decayed as shown in Equation 3.4, so that exploration is favored at the start, while exploitation is preferred in the later stages of learning.

$$\tau(t) = \tau(0)k^t$$

Equation 3.4 Value of τ at time t. k is the decay constant between 0 and 1.

The temperature decay outlined in Equation 3.4 is far from ideal and can lead to slow convergence. Some circumstances require the temperature to be manually tuned with great care (Kaelbling et al. 1996).

3.8 Markov Property

A state is called *Markov* or has the *Markov Property* if it is able to retain all relevant information about its past states. For example, we do not require the complete history of a shuttle cock's path to predict its future path; it suffices to know its current direction and speed. It can be shown that if all the past states have the Markov property then the next state is only dependant on the current state. Iterating this argument, a prediction that is based solely on the knowledge of the current state is just as good as a prediction that is based on the knowledge of all the previous states. From this, it follows that Markov states provide the best possible basis for choosing actions (Sutton & Barto 1998).

3.9 Markov Decision Processes

A learning task that satisfies the Markov property is called a *Markov Decision Process* (MDP). A MDP consists of the following (Kaelbling et al. 1996):

- A set of states S,
- A set of actions A,
- A reward function R: $S \times A \rightarrow \Re$, and
- A state transition function T: $S \times A \rightarrow \Pi(S)$. T(s, a, s') gives the probability of making a transition from state s to state s' using action a.

From the above list we can see that MDPs are important since they can describe the majority of Reinforcement Learning problems (Sutton & Barto 1998). Given any state s and action a, the probability of each possible next state s' is given by:

$$P_{ss'}^{a} = \Pr\{s_{t+1} = s' | s_t = s, a_t = a\}$$

Equation 3.5 Probability of the next state in a MDP.

Similarly, given any current state s, action a and any next state s', the expected value

of the next reward is:

$$R_{ss'}^{a} = E\{r_{t+1} \mid s_t = s, a_t = a, s_{t+1} = s'\}$$

Equation 3.6 Expected value of the next reward in a MDP.

3.10 State-Value Function

A state-value function is a function that estimates the quality of a given state. The quality of a state is defined in terms of the future rewards that can be expected from that state. Given that the expected rewards depend on the actions taken by the agent, the state-value function is defined in terms of the policy used (Sutton & Barto 1998). The value of a state s under policy π is denoted by $V^{\pi}(s)$ and defined as:

$$V^{\pi}(s) = E_{\pi}\{R_t \mid s_t = s\}$$

Equation 3.7 State-value function for policy π.

where E_{π} denotes the expected value given that the agent follows policy π.

3.11 Action-Value Function

For non-deterministic problems where the outcome state of an action is not known, it is not sufficient to use the state-value function. For such problems the action-value function is more appropriate. The action-value function is defined in a similar fashion to the state-value function (Sutton & Barto 1998). The value of taking action a in state s under a policy π is denoted by $Q^{\pi}(s,a)$ and defined as:

$$Q^{\pi}(s,a) = E_{\pi}\{R_t \mid s_t = s, a_t = a\}$$

Equation 3.8 Action-value function for policy π.

Value functions that follow the optimal policy π^* are called optimal and are denoted with *:

$$V^*(s) = \max_\pi V^\pi(s), \quad Q^*(s,a) = \max_\pi Q^\pi(s,a)$$

Equation 3.9 Optimal state and value functions.

It turns out that Q^* can be written in terms of V^* and the immediate return r_{t+1}:

$$Q^*(s,a) = E\{r_{t+1} + \gamma\ V^*(s_{t+1}) \mid s_t = s, a_t = a\}$$

Equation 3.10 Q^* written in terms of V^*.

Some Reinforcement Learning agents use their previous experience to make estimates for value functions V^π and Q^π. For example, for any state s, a simple agent could calculate the average of the rewards received after being in state s. This average will approach $V^\pi(s)$ every time state s is visited. Similarly, $Q^\pi(s,a)$ can be estimated by calculating averages for each action a taken in state s (Sutton & Barto 1998).

It is easy to find the optimal policy from the optimal value functions. Suppose we have obtained V^*, then for each state s, the actions that provide the maximal return are optimal for that state. The optimal policy is one that assigns nonzero probability only to those optimal actions. Therefore with a simple one-step look-ahead of V^* we can find the optimal expected long-term for each state s. If we know Q^* then we don't even need the one-step look-ahead, since $Q^*(s,a)$ provides us with the best action a for any state s. We can see that Q^* allows optimal actions to be selected without any knowledge of the environment's dynamics (Sutton & Barto 1998).

3.12 Temporal Difference Learning

Temporal Difference learning (TD) (Kaelbling et al. 1996; Sutton 1988; Sutton & Barto 1998) is an error-driven method that is without doubt the most central idea of Reinforcement Learning. Although he did not realize it, Samuel was the first to use TD in its basic form (Samuel 1959). Samuel used a learning method which he

called *rote learning*. The algorithm saved a description of each board position encountered together with its backed-up value determined by the minimax search. This meant, that if a position were to occur again as a terminal position of a search, its cached value could be used, which in effect amplified the depth of the search. The algorithm produced slow but continuous improvement that was most effective for opening and endgame play.

Rote learning ensured that the value of a state is closely related to the values of the states that follow it - strongly resembling TD learning. In other respects, the algorithm differs from TD. Samuel's method uses no rewards and has no special treatment of terminal positions of the game. In contrast, TD provides rewards or gives a fixed value to terminal states in order to bind the value function to the true values of the states (Sutton & Barto 1998).

TD can be tabular, where the value of each state is stored in a table; or applied with function approximation techniques such as neural networks. In practice however, tabular TD is only used for simple problems, where the number of states is small and storing their values is feasible.

The TD method uses its experience with policy π to update the estimate of V^{π}. The method updates the estimate of $V(s_t)$ based on what happens after its visit to state s_t. The update can occur immediately at time t + 1, when the method can form a target based on the observed reward r_{t+1} and the estimate $V(s_{t+1})$. Once the target is formed the error term can be calculated and the estimate of $V(s_t)$ updated (Sutton & Barto 1998):

$$\begin{aligned} &\text{Step 1}: \quad \text{target} \leftarrow r_{t+1} + \gamma\ V(s_{t+1}) \\ &\text{Step 2}: \quad \text{error} \leftarrow \text{target} - V(s_t) \\ &\text{Step 3}: \quad V(s_t) \leftarrow V(s_t) + \alpha \cdot \text{error} \end{aligned}$$

Equation 3.11 Updating the estimate for $V(s_t)$ using TD(0).

In the above, α is the learning rate and γ is the discount factor. The learning rate determines how much $V(s_t)$ is updated after each time step; it is set between 0 and 1.

```
Initialize V(s) with random values
For each learning episode
        Initialize state s
        For each step of episode
                Execute action a given by π for s
                Observe reward r and next state s'
                V(s) ← V(s) + α[r + γV(s') - V(s)]
                s ← s'
        Until s is terminal
```

Figure 3.2 Tabular TD(0) for estimating V^{π}.

3.12.1 Advantages of TD methods

TD methods have features that make them well-suited for Reinforcement Learning tasks (Sutton & Barto 1998):

- They learn estimates based on previous estimates (bootstrap). This makes learning faster and more accurate.
- They only need to wait one time-step before learning can take place. This is especially noticeable in tasks with long episodes, where it would be too inefficient to delay learning until the end of the episode.
- They do not require a model of the environment, its reward and next state distributions. This is important for dynamic environments whose model is too hard to compute.
- For any fixed policy π they have been shown to converge to V^{π}. In practice, TD methods have been shown to converge faster than other methods, although there is no theory to support this.

3.12.2 Sarsa: On-policy TD algorithm

When it comes to the trade-off between exploration and exploitation, TD algorithms can be divided into two types: on-policy and off-policy. On-policy algorithms make no distinction between what is being learned and the policy being followed. These algorithms treat exploration as part of the policy being learnt. Off-policy algorithms on the other hand, attempt to learn one strategy (usually the greedy one), while following a strategy that includes exploratory actions (Sutton & Barto 1998).

The difference between on-policy and off-policy algorithms is highlighted in the Cliff Walking task (Figure 3.3). This is a typical gridworld task, where the agent can move in four directions: up, down, left, right. The agent starts at S and must reach goal G. If the agent steps off the world into the region marked as CLIFF it is penalized by a reward of -100 and is sent directly to the start. For all other non-terminal transitions the agent receives a reward of -1. The agent uses ε-greedy action selection with ε set to 0.1 (Sutton & Barto 1998).

Agents that use an on-policy learning algorithm learn to follow the top path. This "safe" path takes into account the possibility of falling off the cliff due to ε-greedy action selection. Off-policy algorithms learn the optimal path, in which the agent travels right along the edge of the cliff. If ε was reduced over time then both types of algorithms would converge to the optimal policy (Sutton & Barto 1998).

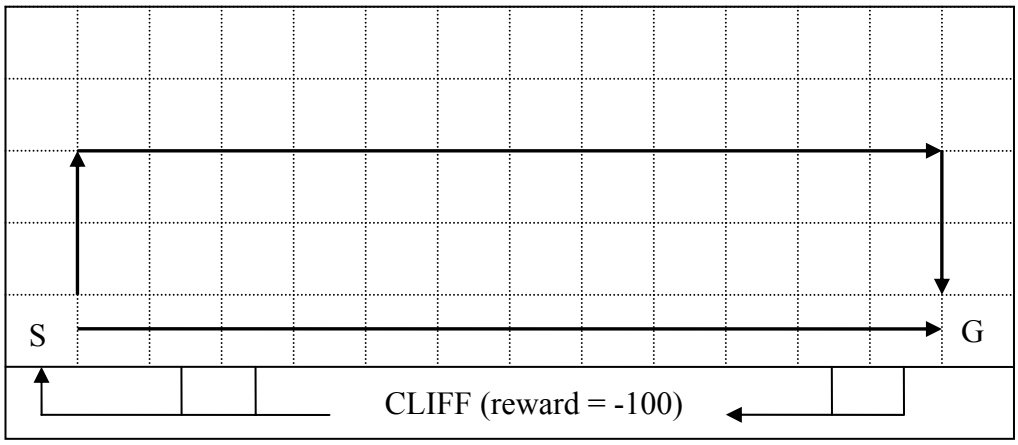


Figure 3.3 Cliff Walking task (Sutton & Barto 1998).

Sarsa is a widely-used on-policy algorithm (Rummery & Niranjan 1994). Sarsa

attempts to learn the action-value function rather than the state-value function. The algorithm estimates $Q^{\pi}(s,a)$ for the current policy π, all states s and actions a. The update of state-action values is performed after every transition from a non-terminal state s_t. If the state that follows (s_{t+1}) is terminal then $Q(s_{t+1}, a_{t+1})$ is defined as zero. We can see where the name Sarsa comes from, as this rule uses every element of (s_t, a_t, r_{t+1}, s_{t+1}, a_{t+1}). Figure 3.4 illustrates the general form of the Sarsa algorithm (Sutton & Barto 1998).

```
Initialize Q(s,a) with random values
For each learning episode
        Initialize state s
        Choose an action a from s to perform non-greedily
        For each step of episode
                Execute action a
                Observe reward r and next state s'
                Choose an action a' from s' non-greedily
                Q(s,a) ← Q(s,a) + α[r + γQ(s',a') - Q(s,a)]
                s ← s'
                a ← a'
        Until s is terminal
```

Figure 3.4 Sarsa algorithm.

3.12.3 Q-Learning: Off-policy TD algorithm

The development of Q-Learning (Watkins 1989; Watkins & Dayan 1992) was one of the biggest breakthroughs in Reinforcement Learning. The one-step Q-Learning update is defined as follows:

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha[r_{t+1} + \gamma \max_a Q(s_{t+1}, a) - Q(s_t, a_t)]$$

Equation 3.12 Updating the estimate for $Q(s_t,a_t)$ using Q-Learning.

In Q-Learning the learned action-value function Q directly approximates the optimal action-value function Q^*, independent of the policy being followed. The policy still has a role of determining which state-action pairs are visited and updated. The details of the exploration strategy do not affect the convergence of the learning algorithm.

For this reason, Q-learning is the most popular algorithm when it comes to learning from delayed rewards in a model-free environment (Kaelbling et al. 1996).

Figure 3.5 shows the general form of the Q-Learning algorithm. Notice that it is nearly identical to Sarsa (Figure 3.4). The only differences are in the highlighted lines: we train on an action that was selected greedily, but then execute an action that was selected non-greedily (Sutton & Barto 1998).

```
Initialize Q(s,a) with random values
For each learning episode
        Initialize state s
        Choose an action a from s to perform non-greedily
        For each step of episode
                Execute action a
                Observe reward r and next state s'
                Choose an action a' from s' greedily
                Q(s,a) ← Q(s,a) + α[r + γQ(s',a') - Q(s,a)]
                Reselect action a' from s' non-greedily
                s ← s'
                a ← a'
        Until s is terminal
```

Figure 3.5 Q-Learning algorithm.

3.13 Eligibility Traces

Almost any TD method such as Sarsa and Q-Learning can be combined with eligibility traces to make it more general. From the practical point of view, an eligibility trace is a record of the occurrence of certain events, such as visiting a particular state and taking a particular action. If the event can undergo learning changes then it is marked as eligible. Whenever a TD error occurs, it is only the eligible states and actions that are responsible for the error and thus only those states and actions get trained (Sutton & Barto 1998).

3.13.1 n-Step TD methods

The backup of simple TD methods is based on information that is just one step ahead. This means that only the next reward and the next state are used for the approximation of previous states. n-Step TD methods are based on n rewards and the estimated value of the n^{th} next states. In contrast, simple TD methods are called *one-step TD methods* (Sutton & Barto 1998). Recall the equation of the expected return (Equation 3.1):

$$R_t = r_{t+1} + \gamma\ r_{t+2} + \gamma^2 r_{t+3} + \ldots + \gamma^{n-1} r_{t+n}$$

Equation 3.13 Expected return again.

We can use Equation 3.13 to calculate the target values for all n in n-step methods. For one-step TD methods the target is the first reward plus the discounted estimated value of the next state ($R_t^{(1)}$ in Equation 3.14). This idea can be extended to the two-step method, where we combine the first and second rewards with the discounted value of the state after the next state ($R_t^{(2)}$ in Equation 3.14). Finally, we can derive the n-step target $R_t^{(n)}$:

$$\begin{aligned} R_t^{(1)} &= r_{t+1} + \gamma\ V(s_{t+1}) \\ R_t^{(2)} &= r_{t+1} + \gamma\ r_{t+2} + \gamma^2 V(s_{t+2}) \\ &\vdots \\ R_t^{(n)} &= R_t + \gamma^n V(s_{t+n}) \end{aligned}$$

Equation 3.14 n-step target calculation.

One-step TD methods use what is called on-line updating. In on-line updating the updates to $V_t(s)$ are performed during the episode, as soon as the increment is computed. n-step methods use off-line updating. This is where the increments are stored in memory and used only once the episode has reached the end. Despite the fact that they converge, n-step TD methods are rarely used in practice. This is mainly because they are difficult to implement (Sutton & Barto 1998).

3.13.2 TD(λ)

Backups can be done not just toward any n-step return, but also toward any average of n-step returns. For example, we can perform a backup which is half of a two-step return and half of a four-step return:

$$R_t^{ave} = \frac{1}{2} R_t^{(2)} + \frac{1}{2} R_t^{(4)}$$

Equation 3.15 Average target backup.

Any set of returns can be averaged in this way, as long as each return is positive and the sum of coefficients is exactly 1. The TD(λ) algorithm can be described as one particular way of averaging n-step backups. The TD(λ) average contains all n-step backups, where each backup is weighted proportional to λ^{n-1}. Finally, the normalization factor of (1-λ) ensures that all weights sum to 1. The resulting backup is called the *λ-return* (Sutton & Barto 1998):

$$R_t^{\lambda} = (1-\lambda)\sum_{n=1}^{\infty} \lambda^{n-1} R_t^{(n)}$$

Equation 3.16 λ-return.

In Equation 3.16 the one-step return is given the largest weight of (1-λ). The two-step return is given (1-λ)λ and so on. Thus the weight of each subsequent return diminishes by λ. Once a terminal state T is reached, all subsequent n-step returns are equal to R_t (Sutton & Barto 1998). Separating this from Equation 3.16 gives the following:

$$R_t^{\lambda} = (1-\lambda)\sum_{n=1}^{T} \lambda^{n-1} R_t^{(n)} + \lambda^{T} R_t$$

Equation 3.17 Detailed version of the λ-return.

We can use Equation 3.17 to highlight the effects of λ. When λ=1, the sum goes to 0 and the remaining term becomes R_t. If λ=0, then λ-return becomes $R_t^{(1)}$, which is the same as the one-step TD(0) method.

In TD(λ) the eligibility trace for state s at time t is denoted $e_t(s)$. On each step, the

eligibility traces for all states decay by γλ, while the eligibility trace for the state being visited is increment by 1 (Sutton & Barto 1998):

$$e_t(s) = \begin{cases} \gamma\lambda\, e_{t-1}(s) + 1, & \text{if s is the current state} \\ \gamma\lambda\, e_{t-1}(s), & \text{otherwise} \end{cases}$$

Equation 3.18 Eligibility trace update (accumulating trace).

This type of eligibility trace is called an *accumulating trace*, because it accumulates each time the state is visited and then decays gradually when the state is not visited. There is also another type of eligibility trace called the *replacing trace*. In replacing trace, every time a state is revisited, its trace is reset to 1:

$$e_t(s) = \begin{cases} 1, & \text{if s is the current state} \\ \gamma\lambda\, e_{t-1}(s), & \text{otherwise} \end{cases}$$

Equation 3.19 Eligibility trace update (replacing trace).

At each step, the traces record which states have been recently visited. This information directly determines which states are eligible to undergo learning changes. The global TD error signal triggers proportional updates to all states with nonzero traces (Sutton & Barto 1998). So we now have:

$$V(s_t) \leftarrow V(s_t) + \alpha \cdot \text{error} \cdot e_t(s)$$

Equation 3.20 Updating the estimate for $V(s_t)$ using TD(λ).

Below is the outline of the TD(λ) algorithm for state-value prediction:

```
Initialize V(s) with random values
Set all eligibility traces e(s) to 0
For each learning episode
        Initialize state s
        For each step of episode
                Choose an action a from s to perform non-greedily
                Execute action a
                Observe reward r and next state s'
                error ← r + γV(s') – V(s)
                e(s) ← e(s) + 1
                For all s
                        V(s) ← V(s) + α·error·e(s)
                        e(s) ← γ·λ·e(s)
                s ← s'
        Until s is terminal
```

Figure 3.6 TD(λ) algorithm.

From Figure 3.6 it may seem that TD(λ) is computationally expensive to implement, because for each state we need to update both its value estimate and eligibility trace. However, only recently-visited states have eligibility traces significantly greater than zero; for most other states the eligibility traces are close to zero. Therefore the convention is to keep track and update only the few states with non-zero traces. Using this trick, the computational expense of using eligibility traces is reduced to just a few times that of one-step methods. In fact if eligibility traces are used with neural networks and backpropagation, then the expense is merely double (Sutton & Barto 1998).

At each state s_{t+1} we find the current TD error and assign it backwards to each prior state according to its eligibility trace. When $\lambda=0$ all traces are zero except for the previous state s_t. Thus in TD(0) only the previous state is updated by the TD error. As λ increases then more and more past states are held responsible for the TD error. However the earlier states are changed less, because their eligibility trace is smaller. When λ reaches 1 then the credit given to earlier states falls only by γ per step (Sutton & Barto 1998).

3.14 Function approximation

For most real-world problems it is impossible to store state-values in a look-up-table because there are simply too many possible states. For example, Othello has a state space of 10^{28}. For such tasks as Othello, function approximation must be used to estimate the expected values. One commonly used form of function approximation is the neural network which is described in detail in the next chapter.

Chapter 4 Neural Networks

4.1 Biological Brain

The biological brain is the most powerful piece of 'machinery' known to mankind. At one single point in time, the brain is able to process vast amounts of sensory input, such as smell, sound, vision, taste and touch, as well as bodily information such as temperature, pulse and blood pressure. After processing all this information, the brain must make complex decisions and take appropriate actions by sending control signals to each and every part of the body.

The brain happens to be one of the most studied organs and yet it is the least understood. Nevertheless, we are able to describe its operation at a low level. We know that the brain contains approximately 10^{10} basic units called neurons, where each neuron is connected to 10^4 others (Beale & Jackson 1992).

Just like any machine a neuron accepts a number of inputs. The neuron becomes activated when enough of these inputs are received at once, otherwise it remains inactive. The body of the neuron is called the soma (Figure 4.1). The soma is attached to strands called dendrites. It is through dendrites that the neuron receives inputs. The dendrites combine inputs similar to a summation method. Another attachment to the soma is the axon. Unlike the dendrite, the axon is electrically active and serves as the output channel of the neuron. The axon produces a voltage pulse when a certain critical threshold is reached. The axon terminates in a special contact called the synapse (Figure 4.1). The synapse serves as a chemical gate between the axon and the dendrite of another neuron; opening when its potential is raised enough by the axon's voltage pulse. Some synapses excite the dendrite they affect, whilst others serve to inhibit it. In a mathematical sense, this is equivalent to altering the local potential of the dendrite from positive to negative respectively (Beale & Jackson 1992).

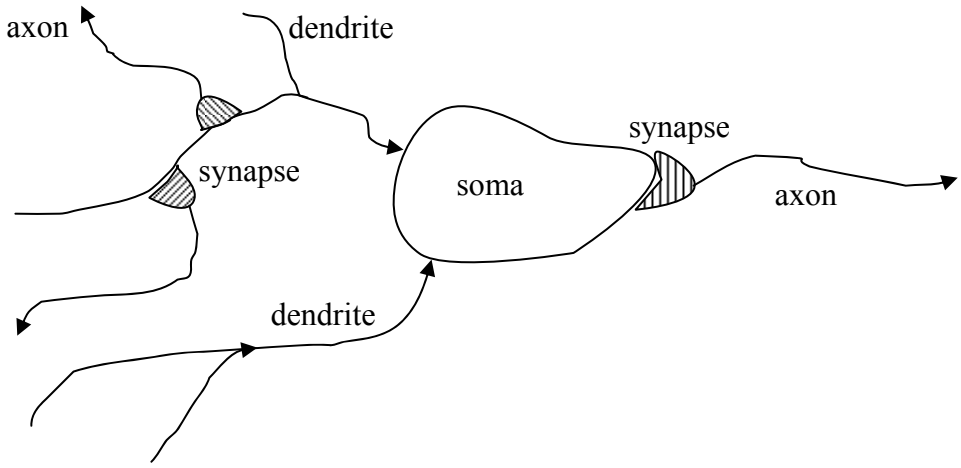


Figure 4.1 Structure of the biological neuron (Beale & Jackson 1992, p. 6).

4.2 Artificial Brain

Computer scientists realised a long time ago the power of the biological brain. If the biological brain can be successfully modelled on the computer than perhaps its power can be extracted and used in a controlled manner. Unfortunately, we cannot yet model the complete structure of the brain, since we do not fully understand how it works. However we can model its major features in the hope that they will lead to the overall structure.

Although the brain is a very complex structure it can be viewed as a highly interconnected network of relatively simple processing elements (Beale & Jackson 1992). As we find in the following sections, this is the essential feature behind current artificial neural networks

4.3 Artificial Neuron

We begin our model by modelling the biological neuron. In its most basic form, the biological neuron combines the inputs received through dendrites and produces an output whenever the combined input exceeds a certain threshold value. The

connections between neurons involve junctions called synapses. These junctions alter the effectiveness with which signals between neurons are transmitted. Some synapses allow large information flow, while others allow very little throughput (Beale & Jackson 1992).

The artificial neuron must capture all of these attributes. The synapse can be modelled as a multiplicative weight. A larger weight corresponds to a synapse that can transfer more information; similarly a smaller weight indicates a weaker synapse. The output of the neuron depends solely on the inputs and is activated only when enough inputs are turned on. The inputs can be combined as a weighted sum (Beale & Jackson 1992). So if there are n inputs: x_1, x_2, x_3, …, x_n and n weights associated with those inputs: w_1, w_2, w_3, …, w_n then the total input becomes:

$$\text{total input} = w_1x_1 + w_2x_2 + w_3x_3 + \cdots + w_nx_n = \sum_{i=1}^{n} w_ix_i$$

Equation 4.1 Total input of the neuron.

The total input is then compared to a certain threshold value θ. If it is greater than θ then the output is 1, otherwise the output is 0. The threshold comparison can be accomplished with a step function:

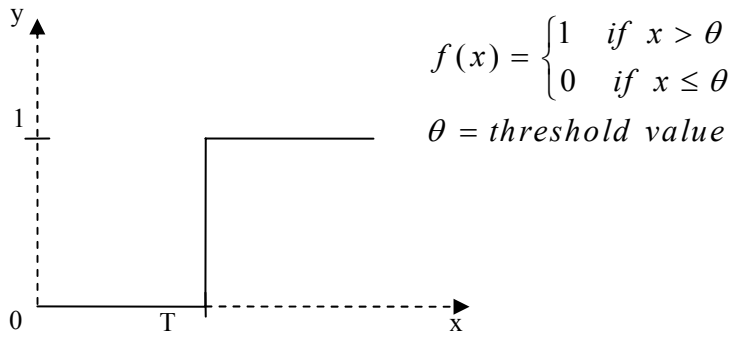


Figure 4.2 Step function used in the neuron.

Notice that in Figure 4.2 the threshold value adds an offset on the x-axis. We can remove this offset by subtracting θ from the original weighted sum. Another way of achieving the same effect is to introduce another input x_0 fixed at +1 whose weight w_0 is set to -θ. Below is the basic model of the artificial neuron:

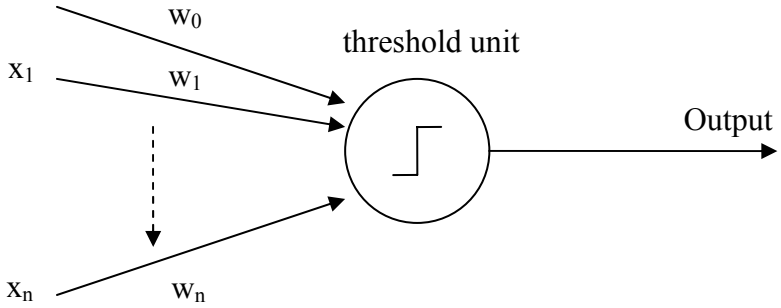


Figure 4.3 Basic model of a neuron (Beale & Jackson 1992, p. 44).

The model of the neuron shown in Figure 4.3 was proposed back in 1943 by McCulloch and Pitts, who had studied the behaviour of the neurons in the brain (McCulloch & Pitts 1943). In 1962 Rosenblatt connected model neurons into a simple network, which he called the perceptron (Rosenblatt 1962).

4.4 Learning with perceptrons

It is not enough to simply connect the model neurons and expect them to do something useful. The networks need to be told which outputs are desirable and which outputs must be avoided. In other words, we need to able to train the network.

Consider the following scenario. Suppose we want to train our network in classifying two classes: A and B. We would like the network to output 1 if the object of consideration belongs to class A; and 0 if it belongs to class B. If the network produces the correct classification then we do not alter since the model has been successful. If the network produces 0 when A is shown, then we want to increase the weighted sum so that next time the network produces the correct result. Similarly, if it produces 1 when B is shown, then we want to decrease the weighted sum. We can increase and decrease the weighted sum by modifying the weights of the active inputs. Notice that there is no point in modifying weights of inactive inputs since they do not affect the result and modifying them may only worsen the outcome. This learning rule is a variant on that proposed by Hebb in 1949, and is therefore called Hebbian learning (Hebb 1949). Figure 4.4 summarizes the above in an algorithm:

Each training example is a pair (X, t), where X is the set of inputs x_0, x_1, x_2 ,..., x_n and t is the target output. η is the learning rate between 0 and 1.

Setup the bias input: set x_0 to 1.
Set all other weights to random values (usually between -0.5 and 0.5)
Repeat until termination condition is met
    Initialize each Δw_i to 0
    For each training example (X, t)
        Calculate the actual output: $o = f(\text{sum}(w_i x_i))$
        Calculate error: $\Delta w_i \leftarrow t - o$
        For each unit weight: $w_i \leftarrow w_i + \eta \Delta w_i x_i$

Figure 4.4 Perceptron learning algorithm (Mitchell 1997).

Note that in the above algorithm the termination condition is not explicitly specified. This is because terminating conditions are highly dependent on the problems being learned. Terminating conditions include: halting after a fixed number iterations, halting once the error of the training examples falls below a certain threshold, halting once the error on a separate validation set meets some criterion. We have to be particularly careful if we chose to halt after a fixed number of iterations. Halting too early can fail to reduce the error sufficiently, while too many iterations can lead to overfitting the training data (Mitchell 1997).

We now turn our attention to the actual algorithm. The Δw_i term allows large weight changes when the actual output is far from the desired output and small changes when the two values are close, thus enabling more accurate convergence of weights. This error term is called the Widrow-Hoff delta, named after its inventors (Widrow & Hoff 1960). Notice that the weights remain unchanged if the network produces a correct output, i.e. $\Delta w_i = 0$ when $t = o$. Widrow referred to these neuron units as adaptive linear neurons (ADALINEs). When many such units were connected they became many-ADALINE or simply MADALINE.

We now turn our attention to the learning abilities of the perceptron. Consider Figure 4.5. In general, if a point has a high x value and low y value then it belongs to the circular class; if it has a low x value and a high y value then it belongs to the square class. The goal of the perceptron is to find a straight line L_n that can separate the two classes, such that the space on each side of the line contains objects from only one class.

At the start of training the perceptron is initialized with random weights. Its first estimate L_1 is not very good as it separates the space into an area with 3 squares and 6 circles, and an area with 5 squares and 2 circles. The learning algorithm ensures that the weights are altered to reduce the error. As the learning proceeds, the separating line tends closer towards the ideal L_n, reaching it after n steps.

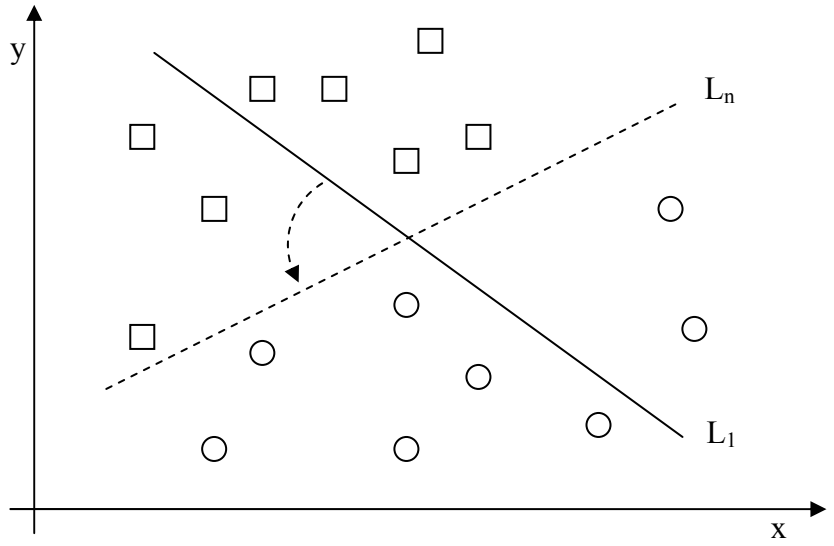


Figure 4.5 Evolution of the classification line L_n from a random orientation L_1 (Beale & Jackson 1992, p. 53).

Rosenblatt showed that given it is possible to classify a series of inputs with a linear decision boundary; the perceptron network will find that classification. He proved that the separating line produced by the perceptron will eventually align itself with the ideal line and will not oscillate around it indefinitely (Rosenblatt 1962).

4.5 Limitations of perceptrons

It can be shown that perceptrons can only classify classes that are linearly separable; these are the classes that can be separated by a single straight line. Unfortunately there exist many problems that are not linearly separable. One of the simplest such problems is called the XOR problem. The XOR function returns true if the inputs are different and false if they are the same:

A	B	A XOR B
False	False	False
False	True	True
True	False	True
True	True	False

Table 4.1 XOR logical function.

Consider the graphical representation of the XOR problem (Figure 4.6). Clearly it is not possible to draw a single straight line that separates squares from circles. Therefore the perceptron will not be able to solve this problem, or any such linearly inseparable problem. This fact, demonstrated by Minsky and Papert in 1969, was regarded as a mortal blow to the area of neural networks (Minsky & Papert 1969). As a result, the majority of the scientific community gave up on the idea and moved on to more promising areas (Beale & Jackson 1992).

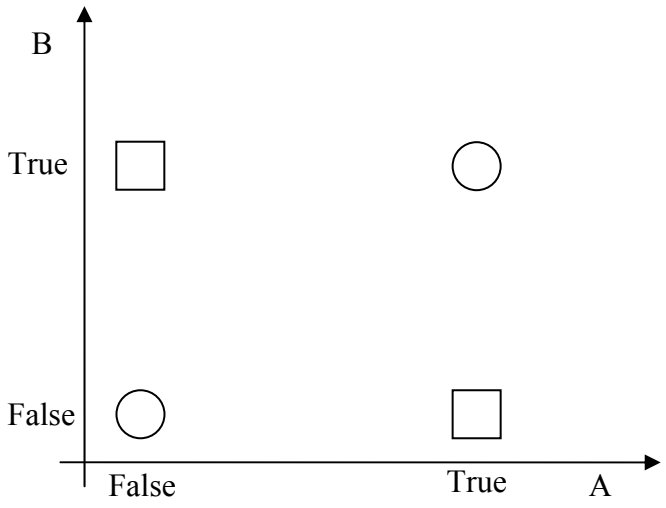


Figure 4.6 Graphical representation of the XOR problem.

4.6 Multilayer Perceptron

Minsky and Papert showed that the key to solving the XOR problem lies in using

more than one layer of perceptrons, i.e. the Multilayer Perceptron, also known as MLP. In the MLP the output of the first layer of perceptrons is treated as the input to the second layer. This way the perceptrons in the first layer can be made to recognize the linearly separable areas of the input. The second layer combines this information to produce the final classification. Unfortunately, for a long period of time researchers were unable to find a method to train MLPs. In 1986 McClelland and Rumelhart breathed new life into the area by introducing their revolutionary training method - backpropagation (McClelland & Rumelhart 1986).

The Multilayer Perceptron is shown in Figure 4.7. The new model has three layers: input layer, hidden layer, output layer. The input layer is the distributor of inputs and hence does not need a threshold function. Each unit in the hidden and output layer is a perceptron that uses a sigmoid threshold function, instead of a simple step function.

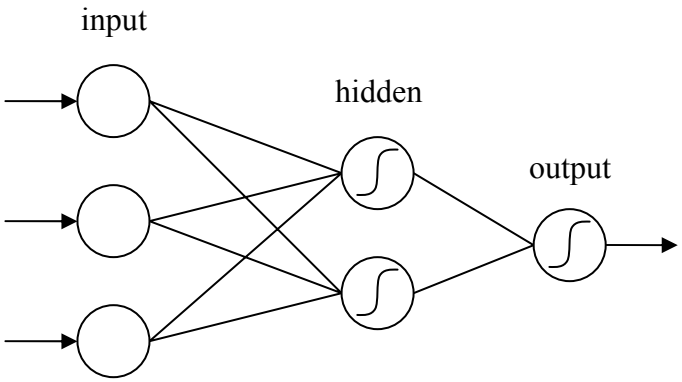
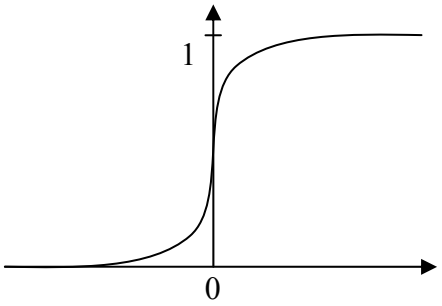


Figure 4.7 Multilayer Perceptron.

The problem in using the step function lies in the difficulty of training the weights between the input and hidden layer. The fact that the step function has a discontinuous derivative makes it unsuitable for gradient descent used by MLPs (Mitchell 1997).

4.6.1 Sigmoid function

The sigmoid function is a smoothed out version of the step function:



$$f(x) = \frac{1}{(1 + e^{-kx})}$$

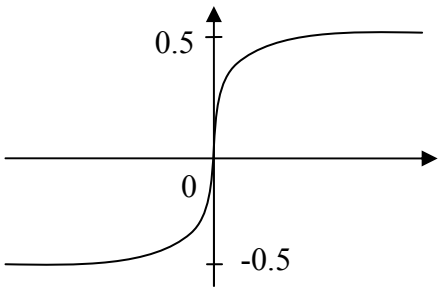
Figure 4.8 Asymmetric sigmoid function.

Although the sigmoid function behaves similarly to the step function at its extreme ends, in the centre it produces a far greater range of values. In the equation, k is a positive constant that controls the spread of the function, as k increases, the function approaches the step function. Although any continuous differential function could be used, the sigmoid is widely used because its derivative can be easily calculated and written in terms of the function itself:

$$f'(x) = \frac{ke^{-kx}}{(1 + e^{-kx})^2} = k \cdot \frac{1}{(1 + e^{-kx})} \cdot \frac{(1 + e^{-kx}) - 1}{(1 + e^{-kx})}$$
$$f'(x) = kf(x)(1 - f(x))$$

Equation 4.2 Derivative of the sigmoid function.

The sigmoid function in Figure 4.8 is called asymmetric, because it is not symmetric around the x axis. In some circumstances it is convenient to use the symmetric sigmoid function shown in Figure 4.9. The derivative of the symmetric sigmoid function is also easy to calculate.



$$f(x) = \frac{1}{(1 + e^{-kx})} - 0.5$$
$$f'(x) = 0.25 - kf^2(x)$$

Figure 4.9 Symmetric sigmoid function.

4.6.2 Backpropagation

The learning algorithm of the MLP is more complex than that of the simple perceptron. The reason is that there are now two layers of weights that need to be trained. Each unit must be adjusted so that it reduces the value of the error function. This is easily achieved for the units in the output layer, since we already know their desired output. However this poses a greater problem for units in the hidden layer. In 1986 Rumelhart, McClelland and Williams suggested a method for training units in the hidden layer, which they called "backpropagation rule" (McClelland & Rumelhart 1986; Rumelhart et al. 1986). It must be noted that similar methods have been proposed earlier by Parker and also Werbos (Parker 1985; Werbos 1974).

Backpropagation relies on the fact that the weights of input and hidden nodes should be adjusted in proportion to the error of the units to which they are connected. Therefore by backpropagating the errors from subsequent layers it is possible to adjust the weights correctly. The standard MLP learning algorithm is given below:

Each training example is a pair (X, T), where X is the set of inputs $x_0, x_1, x_2, \ldots, x_n$ *and T is the set of target output values* $t_0, t_1, \ldots, t_m$. η *is the learning rate between 0 and 1.* x_{ji} *is the input from unit i to unit j.* w_{ji} *is the weight from unit i to unit j.*

Setup the bias input: set x_0 to 1.
Set all other weights to random values (usually between -0.5 and 0.5)
Repeat until termination condition is met
    For each training example (X, T) presented in random order
        Propagate the input forward through the network: Based on input X compute output o_u for every unit u
        Propagate the errors backwards through the network:
            For each output unit k: $\sigma_k \leftarrow o_k(1-o_k)(t_k-o_k)$
            For each hidden unit h: $\sigma_h \leftarrow o_h(1-o_h).\text{sum}(w_{kh}\sigma_k)$
            For each network weight: $w_{ji} \leftarrow w_{ji} + \eta\sigma_j x_{ji}$

Figure 4.10 MLP learning algorithm (Mitchell 1997).

4.6.3 MLP strengths

The MLP succeeds on the XOR problem by using two straight lines to separate the

two classes (Figure 4.11). It turns out that the number of bounding lines that an MLP can produce is equal to the number of hidden units used. This way any convex region such as those in Figure 4.12 a. can be recognized. If we add another hidden layer then we are able to recognize any combination of convex regions (Figure 4.12 b.).

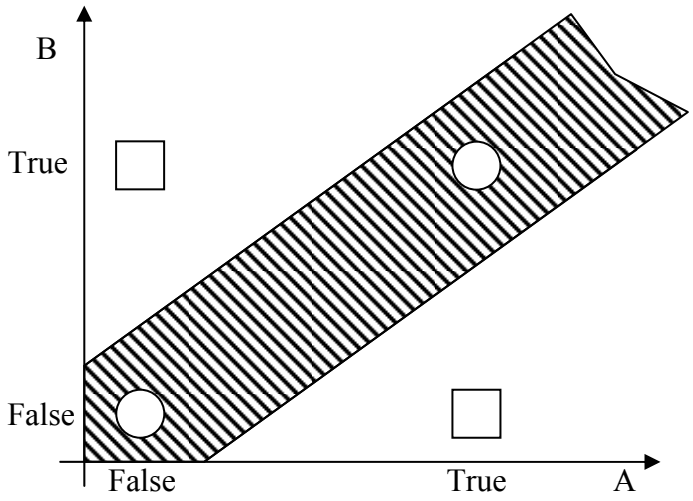


Figure 4.11 XOR problem solved by MLP.

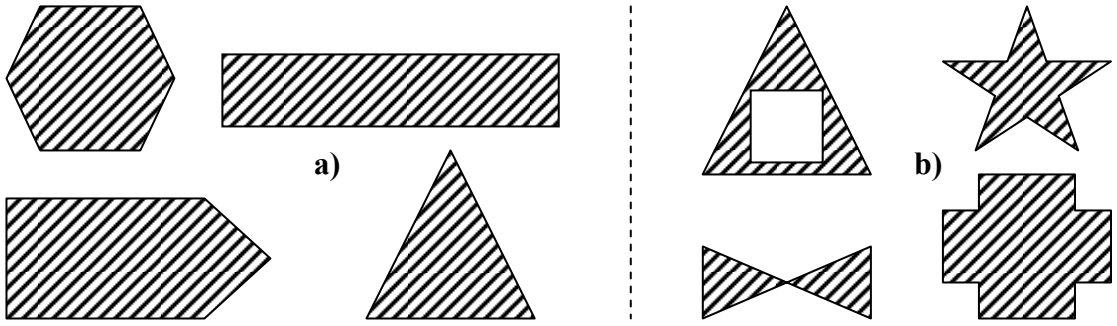


Figure 4.12 a) Examples of convex regions. b) Examples of arbitrary regions formed by the combination of various complex regions (Beale & Jackson 1992, p. 85).

From Figure 4.12 b. it is clear that a 4-layer MLP can recognize any arbitrary shape and can separate any number of classes. This means that we never need to use more than 2 layers of hidden units (this fact is referred to as the Kolmogorov theorem).

The other major strength of MLPs is their ability to generalize. This means that they can classify patterns that have not been previously presented. They achieve this by learning which features of the input are important. An unknown pattern is classified based on how well its distinguishing features match those of others. This generalization ability allows MLPs to perform well in real-world tasks where

nonflexible methods like expert systems fail (Beale & Jackson 1992).

Due to their highly distributed structure, with each node contributing to the overall result, MLPs are reasonably fault-tolerant. Even if some of the nodes become faulty, they will not significantly affect the overall result, since their faults will be smoothed out by other nodes (Beale & Jackson 1992).

An interesting property of MLPs is their ability to discover useful intermediate representations at the hidden unit layers. The network chooses an intermediate representation such that the overall error is minimized. Sometimes, this may lead to intermediate representations that are not explicit from the input, but do well at capturing its properties that are relevant to the target function (Mitchell 1997).

4.6.4 MLP weaknesses

Occasionally MLPs get stuck in local minima from which they cannot escape. This occurs because the network is short-sighted with respect to the error function and moves simply in the direction of greatest descent. When the network reaches the bottom of a well it cannot move any further, because there is no direction to move in order to reduce the error. It is possible to minimize the occurrence of such situations:

- Progressively reducing the learning rate. If η is initialized as a large value (close to 1) then the network takes large steps towards the optimal solution, thus avoiding local minima. As η is reduced the network begins to locate and settle into deeper minima, which are likely to be global (Beale & Jackson 1992).

- Changing the network structure. Local minima occur when two or more disjoint classes are categorized as the same class. This could be the result of a poor internal representation. Internal representation can be improved by adding new hidden nodes or by changing the input format (Beale & Jackson 1992). A network with n weights can represent an n-dimensional error surface. Thus if the method is in a local minimum with respect to one weight then it is

not necessarily in a local minimum with respect to other weights. So, addition of a weight acts as an extra escape route for the gradient descent algorithm (Mitchell 1997).

- Introducing the momentum term α. The effect of α is to produce large changes in weight if the previous changes have been large, and small changes if they have been small. Momentum can also speed up convergence along shallow gradients (Beale & Jackson 1992).

- Adding noise to the input. Random noise can cause the gradient descent algorithm to circumvent the path of steepest descent, which is often enough to escape from a local minimum. This approach is simple and does not noticeably slow down the gradient descent algorithm (Beale & Jackson 1992).

- Train multiple networks with different random weights for the same data set. This is likely to lead to produce networks with varying levels of ability. We can then use the network with the best ability. Alternatively, we can combine those networks to form what is known as an ensemble. The output of an ensemble can be either the average of the individual network outputs or based on some kind of a voting system (Mitchell 1997).

Whilst strong at generalisation, MLPs are not so good at extrapolation. When given an unseen pattern that is a mixture of previously classified patterns, the network tends to classify it as an example of the predominant pattern (Mitchell 1997).

One other problem with MLPs is their slow convergence rate. The networks require many iterations through the same set of input patterns before they are able to settle into a stable solution. This is especially the case in problems with a complex error surface. There are ways of improving convergence rates, such as the addition of the momentum term. Another method is to increase the learning rate η. Finally, it is also possible to use the second order derivative of the gradient, which increases the accuracy of the descent. However, this involves additional computational expenses and is therefore rarely used in practice (Beale & Jackson 1992).

4.7 Constructive Architecture

Constructive neural networks differ from MLPs in two significant ways. First of all they have a constructive architecture in which the hidden nodes are not fixed, but are added one at a time as the network learns. Such hidden units respond only to a local region of the input space. Secondly, they have a different learning algorithm in which only one hidden unit is trained at one time. These networks can form compact input representations, while learning easily and rapidly. Constructive neural networks can be divided into two categories based on the activation function of their hidden neurons. Cascade networks use global activation functions, while resource-allocating networks have local responsive activation functions (Vamplew & Ollington 2005b).

Various cascade-style networks have been developed: Cascade-Correlation, Modified Cascade-Correlation, Cascade and Cascade Network (Lahnajarvi et al. 2002). Based on the findings by Lahnajarvi et al., Cascade-Correlation outperforms all other cascade networks on 5 of the 10 benchmark classification and regression tasks. For the purpose of this thesis, Cascade2 has been used, which is a variation of Cascade-Correlation.

4.7.1 Cascade-Correlation

Cascade-Correlation or simply Cascor networks receive their name due to the cascade architecture and the fact that each new hidden node attempts to maximize the correlation between itself and the residual error signal (Fahlman & Lebiere 1990).

The network begins with some predefined number of inputs and output, but no hidden units. All the inputs (including the bias) are connected to the outputs. The output units can use either a linear or a sigmoidal activation function (Fahlman & Lebiere 1990).

The cascading architecture removes the need for backpropagation and hence the units can be trained with the standard perceptron learning algorithm. Cascor usually uses the Quickprop algorithm (Fahlman 1988). Just like backpropagation Quickprop computes partial derivatives, but instead of using gradient descent it updates weights

using a second-order method related to Newton's method. Quickprop consistently outperforms other backprop-like methods on various benchmark problems (Fahlman & Lebiere 1990).

At some point during training the network reaches a period where there is no significant reduction in error. If the length of this period exceeds a certain threshold (patience parameter) then we run the network one last time and measure the produced error. If the error is sufficiently low then we stop, otherwise it must be the case that there must be some residual error that can be eliminated. This reduction in error can be accomplished by adding a new hidden unit. After the unit is added the cycle is repeated until the error is acceptably small or until the maximum number of hidden units is reached (Fahlman & Lebiere 1990).

To create a new hidden unit, we use a pool of candidate units, each with a different set of random initial weights. The candidate units receive connections from all the inputs, as well as from all existing hidden units. However, they do not interact with each other and do not affect the output of the active network. The candidate units are trained by running a number of passes of the training data. Finally the candidate with the best correlation score C (as given by Equation 4.3) is installed into the network (Fahlman & Lebiere 1990).

$$C = \left| \sum_{p} (V_p - V_{av})(E_p - E_{av}) \right|$$

Equation 4.3 Correlation score.

In Equation 4.3 V_p is the output of the candidate unit for pattern p and E_p is the remaining network output error for pattern p. V_{av} and E_{av} are the values of V and E averaged over all patterns. Due to the absolute sign in the equation, a candidate unit only cares about the magnitude of its correlation with the output error, and not about the sign of the correlation. If a hidden unit correlates positively with the error then it will develop a positive output connection weight, attempting to cancel some of the error; otherwise it will develop a negative weight (Fahlman & Lebiere 1990). In order to optimize C, we must compute its partial derivative with respect to each of the input

weights of the candidate unit:

$$\frac{\partial C}{\partial w_i} = \sum_p s(E_p - E_{av}) f'_p I_{i,p}$$

Equation 4.4 Gradient of the correlation.

In Equation 4.4 s is the sign of the correlation between the candidate's value and the output error, f'_p is the derivative is the derivative of the activation function of the candidate unit with respect to the sum of its inputs for pattern p, and $I_{i,p}$ is the input the candidate unit receives from unit i for pattern p (Fahlman & Lebiere 1990).

As the new hidden unit is installed, its input weights are frozen, meaning that only its output weights are trained (Figure 4.13). In terms of MLPs, each new hidden unit adds a one-unit hidden layer to the network. This can lead to powerful high-order feature detectors, but on the downside, can result in deep networks with high fan-in to the hidden units (Fahlman & Lebiere 1990).

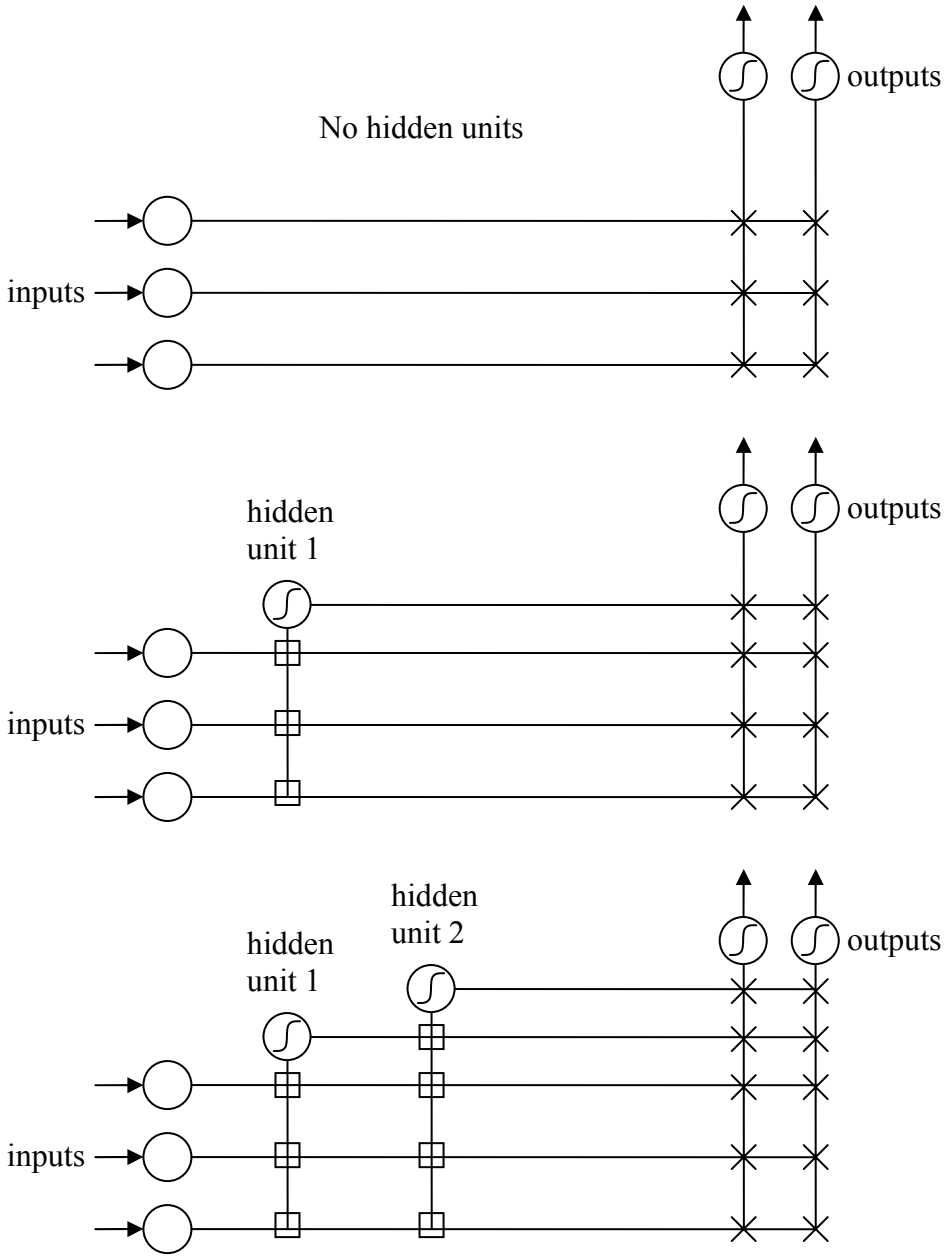


Figure 4.13 Cascor training. Top: initial network with no hidden units. Middle: network after adding one hidden unit. Bottom: network after second hidden unit is added. Boxed connections are frozen, X connections are trained repeatedly (Fahlman & Lebiere 1990).

4.7.2 Cascade networks vs MLPs

Cascor has been shown to equal or outperform fixed-architecture networks on a variety of supervised learning tasks (Fahlman & Lebiere 1990; Waugh 1995).

Fahlman and Lebiere present the following reasons for Cascor's superiority:

- There is no need to have a predefined size, depth and connectivity pattern for the network. With Cascor, all the above parameters will be determined automatically, even though they may not be optimal.

- Cascor learns faster than MLP. MLPs attempt to train all the hidden units at the same time resulting in slow convergence rates (Fahlman and Lebiere refer to this as the "herd effect"). For example, suppose we are learning two sub-tasks A and B. If task A generates a larger error signal then the hidden units tend to concentrate on A and ignore B. Once A is learned, the units start learning B. However, if they all begin to adapt in order to solve B then they will "unlearn" information about task A. In most situations, the "herd" of units will split up and deal with both tasks in parallel, but there may be a long period of indecision before this occurs. In Cascor the hidden units are trained separately. Thus each unit sees a fixed problem and can move decisively towards its solution.

- Cascor is able to build high-order feature detectors without the dramatic slowdown experienced by backpropagation in MLPs.

- Cascor is good for *incremental learning*, in which information is learned bit by bit. Once a hidden unit has learned to recognize a particular feature, its input connections are fixed, meaning that its ability cannot be damaged by new hidden nodes.

4.7.3 Two spirals problem

The classic example where cascade networks outperform MLPs is the two spirals problem proposed by Wieland of MITRE Corp. The problem consists of two sets of points arranged in interlocking spirals that go around the origin three times as shown in Figure 4.14 Left. The goal of the problem is to classify all the points. Lang and Witbrock reported that their best MLP network had three hidden layers with 5 units

each and required 20000 episodes to solve this problem with standard backpropagation (Lang & Witbrock 1988). Cascor with 8 candidate units and a maximum of 15 hidden units required an average of only 1700 episodes to solve this problem (Fahlman & Lebiere 1990).

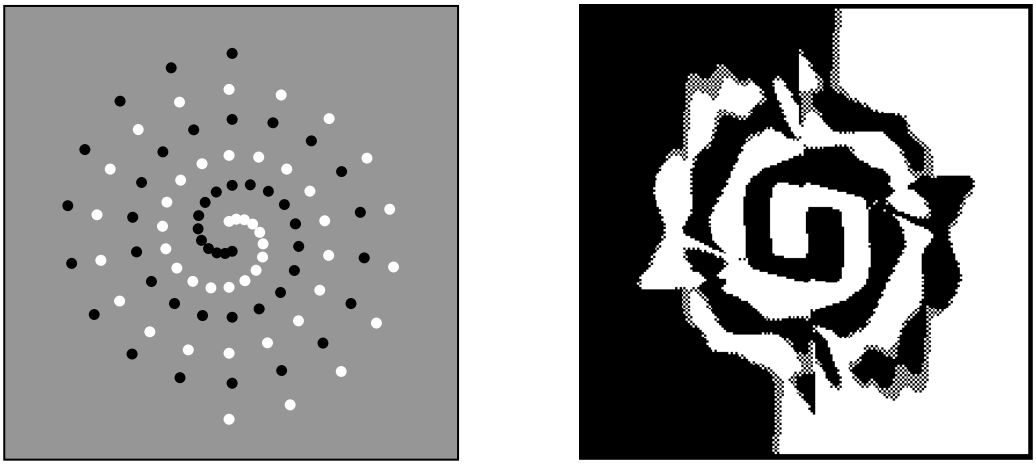


Figure 4.14 Two spirals problem. Left: training data. Right: learned solution (Fahlman & Lebiere 1990).

Fahlman and Lebiere go even further by introducing an alternative measure of learning time called *connection crossings*. A connection crossing is either a forward propagation of the activation value or a back propagation of the error. The best MLPs required 1100 million connection crossings, while Cascor needed just 19 million – a reduction of 50 times. This difference results from Cascor training just one layer of hidden units at a time. The final pattern learned by a Cascor network with 12 hidden units is shown in Figure 4.14 Right. We can see that the output is smooth for the first 1.5 turns of the spirals, but becomes rugged on the outside where the training points are further apart (Fahlman & Lebiere 1990).

4.8 Resource-Allocating network

The resource-allocating network (RAN) was originally proposed by Platt in 1991. The learning algorithm of RAN is somewhat similar to that of Cascor (Platt 1991). The network begins with no hidden neurons and with only the bias weight connected to the output layer. During training, the network allocates new hidden neurons

and connects them to all input neurons. The hidden neurons use a Gaussian activation function which produces a response that is localised to a small region of the state space.

A hidden neuron is added only if both novelty conditions are met. If one of the novel conditions is not met then the output weights and the centres of the hidden nodes are trained using standard gradient descent. In the first condition the distance between the presented input and the centre of the closest hidden neuron response function must exceed a distance threshold. In the second condition the difference between the desired output and the output of the network must exceed the error threshold. The distance threshold is slowly decayed over time. This ensures that initially the network adds neurons with a broad response, while adding more finely-tuned neurons later in training. The centre of the activation function of a new neuron is set to the current input pattern; its width is set to a percentage of the distance to the closest existing neuron. The weight of the new neuron is set to the error between desired and required output, thus eliminating that error (Platt 1991).

RAN was compared to MLP based on its ability to predict the Mackey Glass chaotic time series. Results show that the RAN was comparable to the MLP in accuracy and complexity, but managed to achieve this with 5 times less computation (Platt 1991). More recently, Vamplew and Ollington describe potential advantages and disadvantages for RAN over Cascor specifically in RL problems. First of all, the direct initialization of the hidden neuron parameters is potentially much faster than the training of candidate neurons in Cascor. Secondly, the localised response of the RAN's hidden neurons should give it better generalization abilities, especially in problems with sudden changes in the output function (Vamplew & Ollington 2005b). On the other hand, the RAN's single hidden-layer architecture prevents it from forming higher-level feature detectors that are provided by Cascor. Furthermore the RAN's highly localised nature of hidden neurons forces it to add a much larger number of hidden units. Hence, it is not yet clear which of these architectures is better suited for a given task (Vamplew & Ollington 2005b).

4.9 RL with neural networks

The first known combination of Reinforcement Learning and neural networks was made by Anderson in 1986. Anderson's system used MLPs with Reinforcement Learning algorithm to learn search heuristics. The system was successfully applied to the pole-balancing task and to the Tower of Hanoi puzzle (Anderson 1986). In this section we concentrate on the application of Temporal Difference (TD) learning to neural networks.

4.9.1 TD with Multilayer Perceptrons

There have been many success stories in applying TD with MLPs. Perhaps the most well-known one is that of a Backgammon-playing program called TD-Gammon (Tesauro 1992, 1994, 1995). Initially TD-Gammon had no knowledge of Backgammon strategies, as it was told only the rules of the game and the final outcome – win or a loss. The network received reward of 1 when it won, -1 when it lost and 0 for all other situations. The network used was a standard MLP with one hidden layer and an output indicating the likelihood of the player winning from the current board position.

The network was trained with TD(λ) by playing 1,500,000 games against itself. During the first few thousand games, the network learned a number of elementary strategies. More sophisticated concepts emerged later. In exhibition matches, the latest version of TD-Gammon was able to equal the top human players, losing just 0.02 points per game. Furthermore, TD-Gammon was able to come up with genuinely novel strategies, which have been adopted by top human players.

Crites and Barto applied RL to elevator dispatching. The system examined is a simulated 10-story building with 4 elevator cars. The size of the state space is large ($\sim 10^{22}$) with 2^{18} combinations of call buttons, 2^{40} combinations of car buttons, and 18^4 combinations of positions and directions of the cars. When compared to 8 commercial heuristic systems, the RL system achieved an average improvement of 5% to 15% in system time (Crites & Barto 1996). RL has also been applied in airline revenue management (Gosavi et al. 2002) and marketing (Pednault et al. 2002).

Randlov and Alstrom apply Sarsa(λ) to solve a real-world problem of learning to drive a bicycle (Randlov & Alstrom). Balancing the bicycle is a difficult task to learn, as the agent must take into account the angle and angular velocity of the handle bars, as well as the direction, tilt and velocity of the bicycle. The agent has a choice of just two actions: it can either apply torque to the handle bars or shift its centre of mass. The authors have applied the idea of shaping (borrowed from psychology) to train the agent in a series of increasingly-difficult tasks. The authors note that training through self-play is a form of shaping, because at first the agent plays against a nearly random opponent, thereby solving an easy task. The complexity of the task grows as the agent's playing ability improves. As the result, the agent first learns to balance the bicycle and then later manages to ride it towards a target location (Randlov & Alstrom).

4.9.2 TD with cascade networks

Surprisingly there has been relatively little application of cascade networks to TD learning. The first such application was made by Rivest and Precup for Tic-Tac-Toe (Rivest & Precup 2003) and then later for Car-rental and Backgammon tasks (Bellemare et al. 2004).

The algorithm uses a look-up table (cache) to store the states that have already been found. Before calculating the estimated value V(s) for state s, we first check whether s is in cache. If it is then its cached value is returned. Otherwise, the network is evaluated for s and the value is saved in cache before being returned. When a new target value is computed for state s, it overwrites the current estimate V(s) in the cache. If a state is revisited then its cached value will be updated multiple times according to the TD update rule. After a certain period, the system is consolidated by training the network on the cache data.

The cascade network used was a variation of Cascor. It uses a sibling-descendant pool of candidates, which allows it to add hidden nodes to the same layer, as well as by adding more layers. The results of the Tic-Tac-Toe project showed that Cascor plays equally well or outperforms MLPs against all three built-in opponents:

random, basic and minimax. Interestingly, the structure of the Cascor networks does not become more complex as the strength of the player they are trained against is increased. This suggests that the same network structure is able to learn simple, as well as complex patterns (Rivest & Precup 2003).

A more detailed study of applying Cascor to TD learning was performed by Vamplew and Ollington in 2005. The proposed algorithm (Cascade-Sarsa) differs from Rivest and Precup in three aspects. The algorithm is online, meaning that the network is trained after each interaction with the environment. Secondly, Cascade-Sarsa uses eligibility traces. Finally, it is based on Cascade2, developed by Fahlman as reported in (Prechelt 1997). Although Cascor is effective for classification tasks, it experiences difficulties with regression tasks. This occurs because the correlation term forces hidden unit activations to extreme values, which prevents the network from producing smoothly varying output. Cascade2 differs from Cascor by training candidate units using the residual error as a target output (Vamplew & Ollington 2005a).

Cascade-Sarsa was compared against three other networks, including MLP with a single hidden layer of neurons using asymmetric sigmoid activation functions. The networks were tested on three benchmark RL problems – Acrobot, Mountain-Car and Puddleworld (Vamplew & Ollington 2005b). These problems have been shown to be difficult to learn by MLPs (Boyan & Moore 1995). On Acrobot and Mountain-Car Cascade-Sarsa was significantly more successful than MLP on both on-line and off-line tests. However, Cascade-Sarsa failed on the Puddleworld problem, not even able to match the relatively poor on-line performance of MLP (Vamplew & Ollington 2005b).

To explain this difference in performance we take a closer look at the problems being learnt. In both Acrobot and Mountain-Car tasks the decision boundaries are linear, and hence each boundary can be learnt by a single sigmoidal hidden unit. Naturally Cascade-Sarsa performs well on these tasks, because each candidate node can specialize on a single decision boundary. In contrast the Puddleworld task requires networks to learn localised regions within the input space, which cannot be achieved by a single candidate node used by Cascade-Sarsa (Vamplew & Ollington 2005b).

Chapter 5 Othello

5.1 History of the game

Formally speaking, Othello is a two-person zero-sum deterministic finite board game with perfect information. Two-person zero-sum games are characterized by the fact that either one player wins and the other loses, or neither player wins (tie). It is not possible for both players to win. Deterministic games are games where there are no random events such as the roll of a dice. Othello is a finite game, because there is a finite number of moves: at most 60. Othello has perfect information, because all the game information such as piece position is available to all players.

The exact origins of Othello are not known, although it is believed to have originated from the Chinese game 'Fan Mian', which translates to 'reverse' (hence the alternative name Reversi). Until recently the game has been primarily played in Japan, where it is the second most popular game next to Go.

In 1890's a game called Reversi was being marketed in England by Jaques and Sons of London. This game was very similar to the modern game except for two differences:

1. Each player was given a set number of discs at the start of the game. If a player ran out of discs, his opponent completed the remainder of the game.
2. The game began with an empty board.

In 1975 Goro Hasegawa introduced the modern version of Othello which has been adopted across the world. In the modern version, players take from a central pool of discs and the initial layout of four discs is provided. Othello is fast growing in popularity with major international tournaments held every year. It has also been a popular test-bed for AI research (Buro 1994; Lee & Mahajan 1990; Rosenbloom 1982).

5.2 Rules of the game

Othello is one of those games that take minutes to learn, but years to master. Great expertise is required to play well, since board positions can change drastically in just a few moves, especially towards the end of the game.

The game is played on a chess-like 8 x 8 board with a set of dual-coloured black and white discs. The initial setup of the board contains four discs placed in the centre; 2 white and 2 black (Figure 5.1). The game begins by black making the first move. A move involves placing a disc into an empty square on the board. Legal moves are those moves that result in capture of opponent's pieces. Opponent pieces become captured if they are surrounded, in a straight line, between the disc being played and another disc belonging to the current player (Figure 5.2 Left). Once a disc is captured it is flipped over to the opponent's colour (Figure 5.2 Right). If a player cannot make a legal move then he has to pass and the move is transferred to his opponent. The game finishes when neither player has a legal move, usually when all 64 discs have been placed. At the end of the game all discs are counted and the player with the most discs is declared to be the winner; unless both players have the same number of discs in which case it is a tie.

	A	B	C	D	E	F	G	H
1		c	a	b	b	a	c	
2	c	x					x	c
3	a							a
4	b			○	●			b
5	b			●	○			b
6	a							a
7	c	x					x	c
8		c	a	b	b	a	c	

Figure 5.1 Initial setup of the board with common names of squares shown.

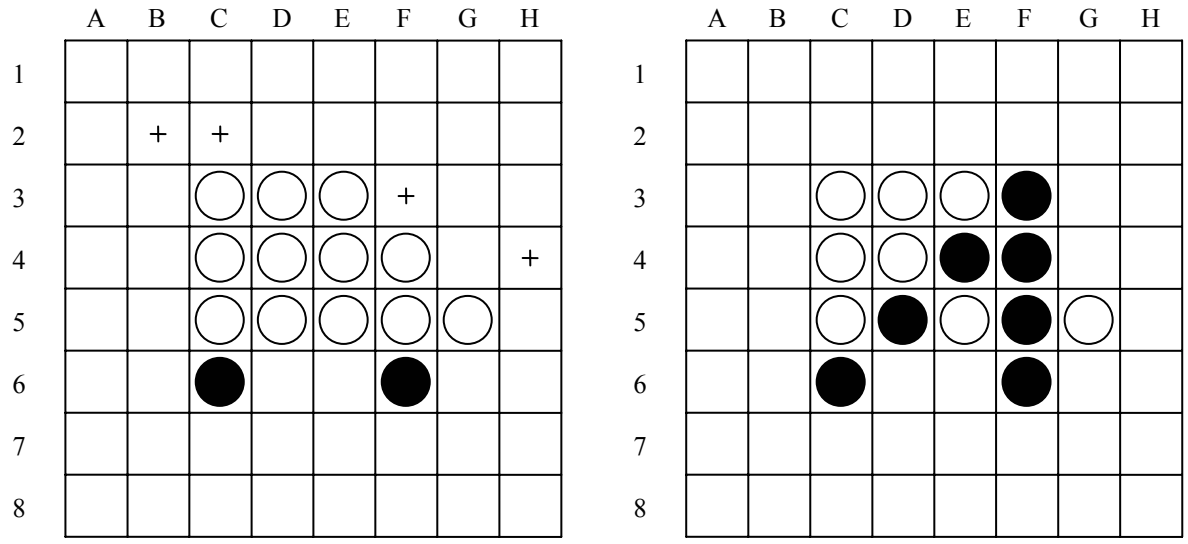


Figure 5.2 Left: Black's move. Legal moves are indicated with +
Right: A black piece is placed in F3 which results in the capture of D5, E4, F4 and F5. Note that E5 is not captured because it is not in 'line of sight' of the new piece at F3.

5.3 Strategies

Othello players use a number of strategies to maximize their chances of winning. Here we discuss some of the common strategies that human and computer players use. We begin with some of the simpler strategies like Disc Difference and move on to more complex strategies such as Mobility.

5.3.1 Disc Difference (Greedy)

The most basic strategy is to maximize the number of one's pieces at every move. This strategy is often used by novice Othello players. Although this strategy is very important in the later stages of the game, it can become disastrous when used earlier in the game. To illustrate this point, consider Figure 5.3 Left. Here it may seem that white is in a winning situation with 59 discs. However, the next four moves (A1, H1, H8, and A8) belong to black and the game changes dramatically with black winning 40 to 24 (Figure 5.3 Right).

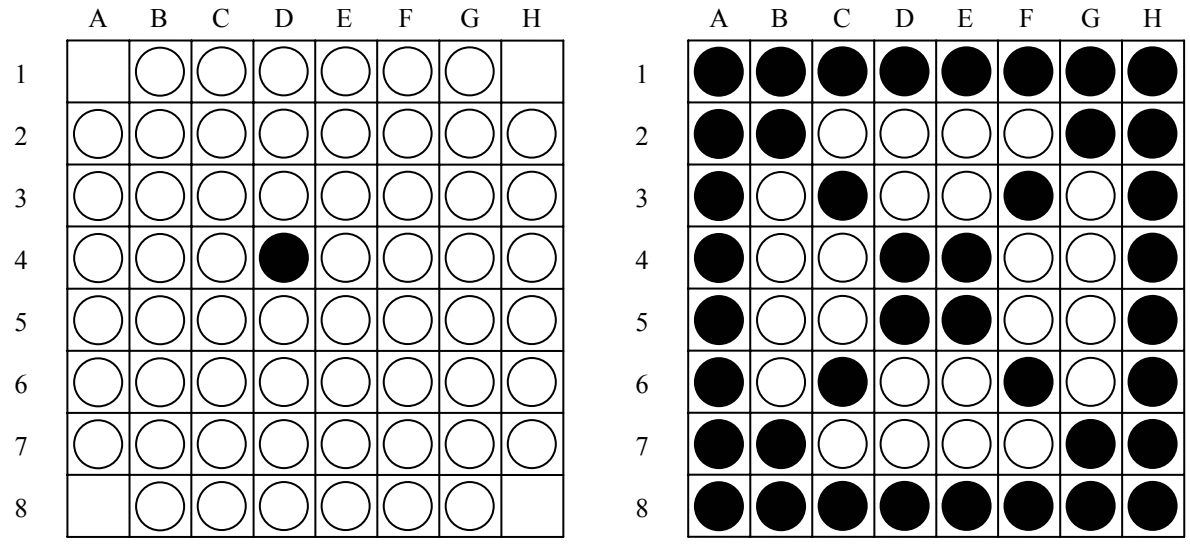


Figure 5.3 Left: White is using the Disc Difference strategy and seems to be in total control of the game. Right: Black captures all four corners, giving him the win with 40 to 24 discs.

5.3.2 Positional

A player using the positional strategy recognizes that some locations on the Othello board are more valuable than others. For example, the corners are very valuable, because a disc placed in a corner cannot be captured. Such discs are called *stable*. Stable discs are important because they are guaranteed to remain to the end of the game and hence add to the total score of their owner. Furthermore, they serve as a means for capturing opponent's non-stable discs. Thus the stability of the player's discs refers to the number of stable discs that the player possesses and their locations. It is often the case that good stability leads to a winning situation; however this does not necessarily imply that controlling all four corners leads to a win (see Figure 5.4 for a counter-example). Discs placed in a and b squares are considered *semi-stable* because they become stable once the player gains control of the corresponding corner. These discs may also help to capture the corner.

The positional player avoids going to the x square, because it usually gives away the adjacent corner and prevents the player from getting that corner himself. Similarly, the positional player is cautious when going to the c square, as that square can also give away corners.

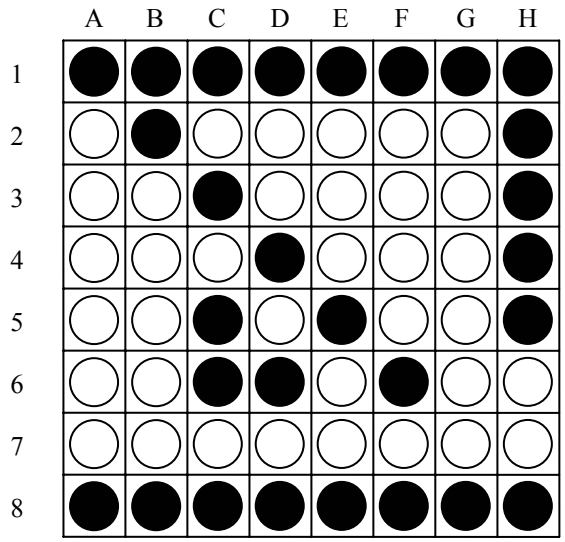


Figure 5.4 A counter-example. Black loses 28 to 36 despite having control of all four corners.

5.3.3 Mobility

Mobility is one of the best strategies in Othello, but is also one of the hardest to master. The mobility of a player refers to the number of legal moves available to that player. Lack of mobility can lead to severe difficulties. A player with poor mobility may be forced to choose 'bad' moves that lead to a detrimental situation (Figure 5.5). On the other hand, a player with good mobility has a large choice of moves, allowing him to direct the game towards an advantageous situation. Mobility is important during the middle part of the game where both players are fighting for a good position, which will provide easy capture of corners.

It is believed that the mobility strategy was discovered in Japan in the mid 1970s. The strategy slowly spread to America and Europe through personal contact between American and Japanese players. It has been shown that games played with mobility strategy were more difficult to recall, suggesting that the mobility strategy is much harder to learn (Billman & Shaman 1990).

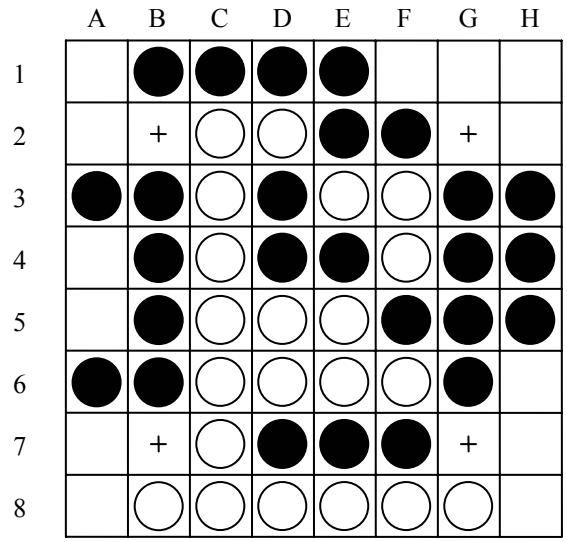


Figure 5.5 An example of poor mobility. Black has only four legal moves (marked with +), as opposed to white who has 11. Furthermore, any move that black chooses will surrender a corner.

5.4 Interesting Properties

Othello has some interesting properties that make it different from other board games. In Othello, every move is important, as it is very hard to recover once a mistake has been made. This is especially true towards the end of the game, where a single move can change the board state dramatically. For example consider the board in Figure 5.6 Left, white to move. If white goes to G2 then black will go to H1 and will win 61-0. However, if white goes to A8 he can win 48-16. Thus a simple choice between two moves can lead to final difference of 93 discs!

The outcome of the game is also highly dependent on whose move it is. Consider the board in Figure 5.6 Right. If black is to move then he can win 51-0 by going to H1. However if white is to move then he can force a 60-4 win, giving a difference of 107 discs.

In 1993 former British Othello Champion Feinstein weakly solved the 6 x 6 version of Othello. It took him 2 weeks of computer time to show that the second player can force a 20-16 win (Feinstein 1993). Nevertheless, it is still questionable whether these findings scale up to an 8 x 8 board.

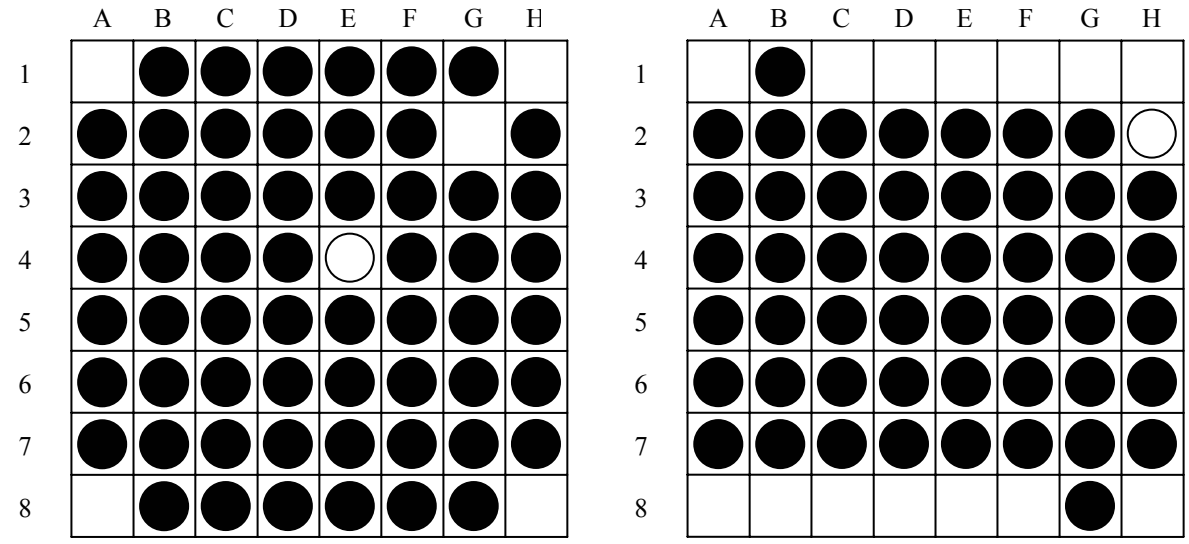


Figure 5.6 Interesting Othello properties. Left: highlights the importance of making a correct move. Right: highlights the importance of having the next move.

Many Othello experts believe that the second player has an advantage in the game. Indeed to ensure that there is no bias towards the second player, professional tournaments require players to play two games against their opponent: one with black and one with white discs. Due to the symmetry of the initial board configuration, all four moves available to the first player are identical. It is the second player who is first to make a "meaningful" move. Furthermore, if both players play well then there will be no passes and the game will end in exactly 60 moves. In this case the second player will go last and have the advantage of taking the few final discs.

5.5 Why Othello?

The first part of this project involved choosing a suitable game to be used for learning. A number of games have been considered for this purpose. These included games that have been studied in the past such as: Backgammon (Tesauro 1992, 1994, 1995), Checkers (Chellapilla & Fogel 1999; Fogel 2000; Samuel 1959), Chess (Beal & Smith 1997; Dazeley 2001; Fogel et al. 2004), Go (Ekker et al. 2004; Richards et al. 1998; Schraudolph et al. 2000) and Othello (Buro 1994; Lee & Mahajan 1990; Rosenbloom 1982); as well as games that have not been deeply researched: Bantumi,

Chinese checkers and Lines. Othello was chosen for the following reasons:

- It is a well-studied game, researched by many authors using a variety of techniques.
- The rules of the game are simple and easy to implement.
- It is easy to implement basic AI opponents, such as a random player and a greedy player.
- From Table 5.1 in section 5.5.4, we can see that Othello has a reasonable complexity. It is not as simple as Checkers and not as complex as Chess or Go.
- Since every game is guaranteed to finish in 60 moves or less, Othello is perfect for rapid training of neural networks.

Ghory identifies four game properties that are crucial for a successful game playing agent that is based on TD-Learning: board smoothness, board divergence, forced exploration, state-space complexity (Ghory 2004).

5.5.1 Board smoothness

It is known that neural networks are better at learning smooth functions. A function f(x) is smooth if a small change in x implies a small change in f(x). Hence, if |A – B| is the absolute difference between two board states A and B then we want a small value in |A – B| to correspond to a small value in |e(A) – e(B)|, where e is the board evaluation function. Board smoothness can be achieved by carefully selecting the right board representation. The most natural way to represent an M x N board is to have MN inputs, where each input stores the contents of a single board square. It is hard to say whether this representation achieves the optimal board smoothness, but it is a popular choice when it comes to representing Othello boards (Deshwal & Kasera 2003; Leouski 1995; Tournavitis 2003; Walker et al. 1994).

5.5.2 Game divergence

The divergence of a board state A is defined as the average difference between A and its child board states B_1, B_2, …, B_n:

$$\text{Divergence}(A) = \frac{\sum_{i=1}^{n} |A - B_i|}{n}$$

Equation 5.1 Divergence of a board state.

Similarly, the divergence of a game can be defined as the average of *Divergence(A)* for all possible board states A. The most optimal divergence of 1 occurs in games such as Connect-Four and Tic-Tac-Toe, where every move only adds a single piece and does not alter any other pieces on the board. To study the divergence of Othello, we made two random players play against each other, recording the divergence at each stage of the game (Figure 5.7). The divergence increases slowly at a constant rate until the 50th move, at which point it starts increasing exponentially. Based on Figure 5.7, the average divergence is 3.25. It must be noted however, that a single move can alter up to 18 discs.

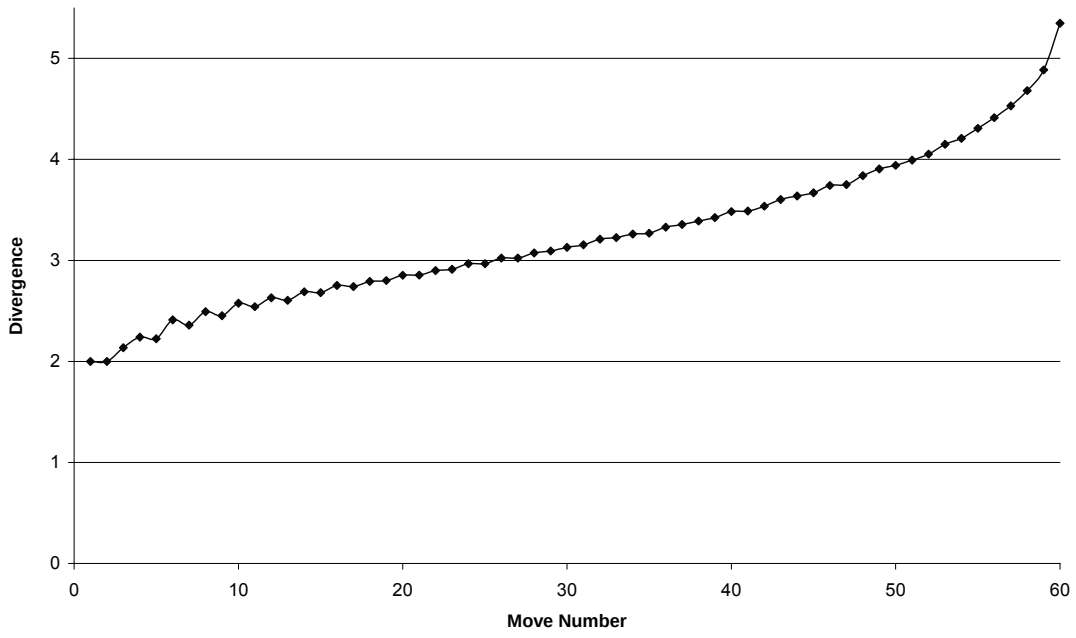


Figure 5.7 Divergence of Othello when the game is played randomly.

5.5.3 Forced exploration

There is a general consensus amongst AI researchers that a learning agent should not always choose the best move available as this will restrict his knowledge of the environment. Occasionally, the agent should choose a weaker move in order to explore new possibilities and hopefully learn a better strategy. Games that have a random element such as Backgammon or Poker are ideal, because they force the agent to explore. Pollack and Blair claim that forced exploration of Backgammon was one of the main reasons for the success of Tesauro's TD-Gammon (Pollack & Blair). However, it is still possible to force exploration in deterministic games by introducing randomness to the move selection strategy.

5.5.4 State-space complexity

State-space complexity is defined as the number of board states that can occur in a game. It is important to know the state-space complexity as it defines the domain of the board evaluation function. Thus the larger the state-space complexity the longer it will take to train the board evaluation function. Van Den Herik et al. (2002) present state-space and game tree complexities of some games (Table 5.1). If we assume that any of Othello's 64 board locations can take three values (black, empty, white) then we can calculate the upper boundary for Othello's state-space complexity to be $3^{64} \approx 3.4 \times 10^{30}$. This value indicates that less than 0.3% of theoretical board states are possible during the actual game. Ghory estimates that it is possible to produce a strong TD-Learning based agent for any game whose state-space complexity is less than 10^{20}. He goes on to claim that the state-space complexity of a game is the single most important factor out of the four game properties discussed (Ghory 2004).

Game	Complexity	
	State-space	Game Tree
Backgammon	10^{20}	10^{144}
Checkers	10^{18}	10^{31}
Chess	10^{46}	10^{123}
Connect-Four	10^{14}	10^{21}
Go (19 x 19)	10^{172}	10^{360}
Othello	10^{28}	10^{58}

Table 5.1 Complexity of some games (Van Den Herik, Uiterwijk & Van Rjiswijck 2002).

In Table 5.1 state-space complexity represents the number of possible board states in the game. For example, in Othello there are 64 board locations where each location can take one of three values, giving approximately $3^{64} \approx 10^{28}$ total states. Game tree complexity represents the total number of nodes in a fully-expanded game tree. From Table 5.1 Othello has a game tree complexity of 10^{58}. Based on that figure, the average branching factor is about 9.26. To check this, we made two random players play against each other, recording the number of available moves for each stage of the game. Figure 5.8 shows the result of that experiment. We can see that the branching factor rises slowly from 4 and peaks at 12 on the 30^{th} move. After that the curve undergoes a sharp decline. The average branching factor experienced by the players is 8.54.

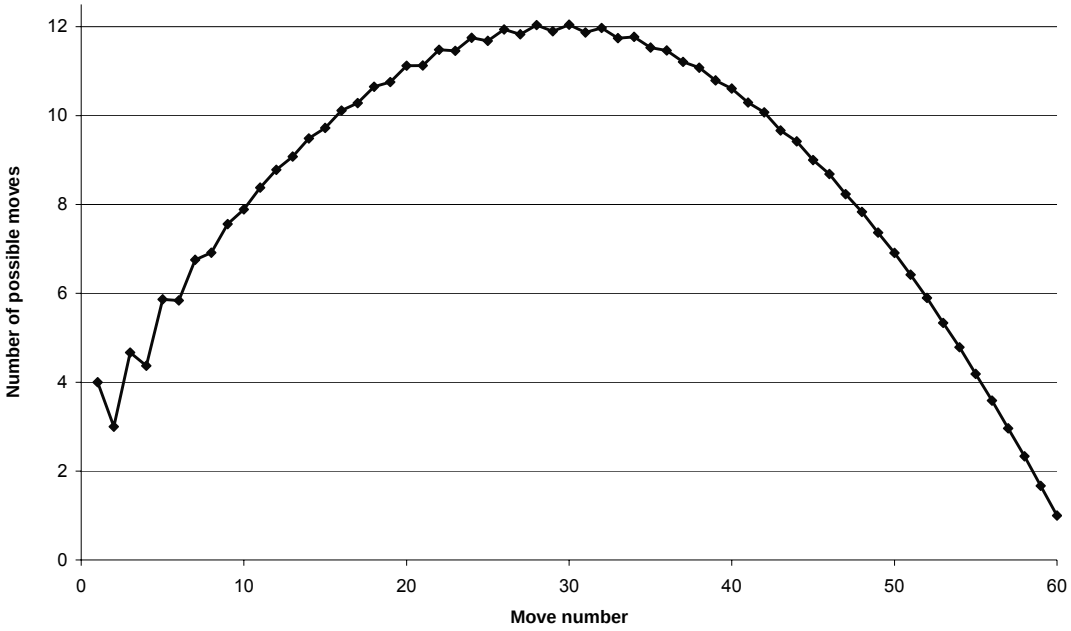


Figure 5.8 Branching factor when the game is played randomly.

5.6 Classic Othello programs

5.6.1 IAGO

IAGO developed by Paul Rosenbloom was the first notable Othello-playing program (Rosenbloom 1982). IAGO was written in the SAIL language and was running on CMU-20C System that had only 512K of primary memory. IAGO won the 1981 Santa Cruz Open Machine Othello Tournament with a perfect 8-0 record.

IAGO is based around the α-β search which uses a hand-crafted evaluation function to select the best available move. IAGO uses iterative deepening to ensure that it can complete moves within the allocated time limits. IAGO's evaluation function for a board position p is a weighted sum of four main components: edge stability, internal stability, current mobility and potential mobility:

$$Eval(p) = ESAC(MoveNumber) \times EdgeStability(p) + 36 \times InternalStability(p) + CMAC(MoveNumber) \times CurrentMobility(p) + 99 \times PotentialMobility(p)$$

Equation 5.2 IAGO's evaluation function

Initially, the coefficients were based on the opinion about relative importance of individual components. Variations of these values were tested and the best-performing variation was chosen. ESAC and CMAC are application coefficients whose values vary with move number, hence reflecting the level of importance of edge stability and current mobility at various stages of the game. Edge stability uses a pre-computed table of all 6561 possible configurations of eight edge squares. Each table element represents the value of a particular edge as viewed by Black who has the next move. Internal stability assigns a value based on the number of stable internal discs. Current mobility refers to the number of moves currently available. Potential mobility is the potential of the current board position to lead to future mobility. This value is calculated by applying three measures: number of frontier discs, number of empty squares adjacent to opponent's discs, and the sum of the number of empty squares adjacent to each of the opponent's discs.

IAGO does not use an opening book and is probably hurt by this. However, IAGO can solve end-game positions. It can find the winning move combination at move 46, as well as the move that maximizes the final disc difference at around move 48.

5.6.2 BILL

The first version of BILL was developed by Lee and Mahajan in 1986. BILL was inspired by the success of IAGO. The latest version of BILL, BILL 3.0 took first place in the 1989 North American Computer Othello Championship and beat Brian Rose, the highest rated American Othello player at the time. In games against IAGO, BILL always wins with only 20% as much computational time (Lee & Mahajan 1990).

BILL uses hash tables to store information from previous searches of iterative deepening. These tables increase the probability that the best descendants of a

move are examined first, hence increasing the amount of pruning. The number of alpha-beta cut-offs is also maximized through the use of zero-window searching. BILL also solves end-game positions. It applies the outcome (win/loss/draw) search with 15 to 18 moves remaining and the exact disc differential search with 13 to 15 moves remaining.

BILL's evaluation function is composed of four features: edge stability, current mobility, potential mobility, and sequence penalty. Just like IAGO, BILL pre-computes a table of all possible edge combinations. Each table element is a combination of 8 edge squares and two x-squares, which the authors believe have a prominent effect on the value of an edge. Current mobility is based on the number of available moves and adjusted upon three factors: whether moves capture or surrender a corner, number of discs flipped and in how many directions, whether flipped discs are internal or on the frontier. Potential mobility captures the likelihood of future move options by counting frontier discs. BILL applies a sequence penalty in order to avoid long sequences of discs, since such sequences often hinder mobility. To form the final evaluation, the above four features are combined using a quadratic discriminant function g(x):

$$g(x) = (x - \mu_{Win})^T \sum_{Win}^{-1} (x - \mu_{Win}) - (x - \mu_{Loss}) \sum_{Loss}^{-1} (x - \mu_{Loss}) + \log\left|\sum_{Win}\right| - \log\left|\sum_{Loss}\right|$$

Equation 5.3 BILL's quadratic discriminant function

Using Bayes' rule the probability of a win from the current position x is given by:

$$P(Win \mid x) = \frac{e^{g(x)}}{e^{g(x)} + 1}$$

Equation 5.4 BILL's evaluation function

5.6.3 LOGISTELLO

LOGISTELLO was developed by Michael Buro as part of his PhD thesis and is currently one of the strongest Othello programs. Between 1993 and 1997

LOGISTELLO dominated all computer Othello tournaments by winning 18 out of a total of 25. In 1997 LOGISTELLO defeated the reigning World Champion Takeshi Murakami by winning 6-0.

Although BILL showed a strong performance, many of its features still relied on manually-tuned parameters. The main idea behind LOGISTELLO is to learn pattern values from sample data – rather than guessing them (Buro 2003). LOGISTELLO treats the game as 13 separate stages: 13-16 discs, 17-20 discs, … , 61-64 discs (for fewer than 13 discs the opening book is used). Each game stage has a different set of weights in the evaluation function. The evaluation function analyses 46 patterns of squares that capture important strategic concepts such as mobility, stability and parity. These patterns include all diagonals longer than 3 squares, all rows, as well as 2x5 and 3x3 corner regions. The evaluation of a particular board approximates the final disc differential by summing up the weights associated with the matching patterns.

Buro made a considerable effort to develop LOGISTELLO's opening book. The opening book is a table of precomputed evaluations for most common game openings. LOGISTELLO has learnt its opening book from 23000 tournament games and evaluations of "best" move alternatives. Furthermore, it continues to learn with every new game and thus is guaranteed not to make the same mistake twice.

To speed up the searching procedure Buro introduced a method called ProbCut. ProbCut prunes sub-trees that are unlikely to yield a good solution. The time saved is used to explore better possibilities (Buro 1995). An extension to ProbCut is Multi-ProbCut, which can prune at different search depths, uses game-stage dependent cut thresholds, and performs shallow check searches (Buro 2003). The combination of these improvements allow LOGISTELLO to perform an outcome end-game search with 22-26 moves remaining and an exact search with 20-24 moves remaining.

5.7 Othello programs using TD

There have been a number of applications of TD to Othello (Cranney 2002; Deshwal & Kasera 2003; Leouski 1995; Tournavitis 2003; van Eck & van Wezel 2004; Walker

et al. 1994). In this section we discuss three applications that are most relevant to this project.

The earliest application of TD to Othello was in 1994 (Walker et al. 1994). The aim of that research was to replicate the results of Tesauro in Othello to form TD-Othello. Unlike Backgammon, Othello is deterministic and noise needs to be added. The authors simulate noise by adding a random value ε_m to the output of the network, with ε_m uniformly distributed in the range [-0.1, 0.1]. In essence this is similar to softmax selection: the moves that were ranked highly before noise was added are also likely to rank highly with noise and hence will be selected; moves that ranked lowly will remain ranked lowly and hence will not be selected. As the result of just 30,000 self-play games the agent was able to win against the Novice (2^{nd}) level player of Windows Reversi (Version 2.03). Furthermore, by providing a one-step look ahead the agent managed to consistently beat Windows Reversi at the Expert (3^{rd}) level, and even had occasional wins at the Master (4^{th}) level. Due to its ability to beat a commercial product, TD-Othello was chosen as a base model to develop the Othello player for this project.

Leouski draws attention to the fact that the Othello board is symmetrical with respect to reflections and rotations (Leouski 1995). This symmetry is incorporated into the network by appropriate weight sharing (Rumelhart et al. 1986) and summing of derivatives. The initial network was able to win 42% against Wystan (Clouse 1992), a program that incorporates several high-level features such as mobility and stability. Leouski then creates an agent that uses three separate networks – one for each stage of the game. This improves the agent's performance considerably as it is able to win 73% against Wystan. The author postulates that the agent's performance can be boosted further by growing the network dynamically as it learns. This is exactly what our project aims to test.

Cranney makes comparisons between self-play and tournament-play learning applied to Othello (Cranney 2002). In tournament-play, multiple agents are trained by playing against each other. The author argues that tournament-play leads to greater exploration, since the networks get to experience a wider range of strategies. However, this also means that tournament-play is less likely to converge to a good

strategy. In conclusion, Cranney shows that self-play outperforms tournament-play for simple input, but when advanced input encoding is introduced, tournament-play gains an evident advantage. In this project we are mainly concerned with the agent's ability to learn from simple inputs and hence self-play was chosen as the training method.

Chapter 6 Method

6.1 Training environment

The program used in this project can be divided into two main environments: training and testing. In training, the following were taken into account: various input schemes, action selection strategies, reward function, parameters common to all neural networks, those specific to the fixed architecture and those specific to the constructive architecture. In testing, consideration was given to the following: built-in opponents, ways of measuring learning and other potentially useful features.

6.1.1 Input representation

In Reinforcement Learning problems the success of the learning agent is highly dependent upon the input representation used. It is important to remember that we are teaching a computer agent to learn, not a human agent. Inputs that can be easily interpreted by a human, such as a picture of a fish, are not necessarily easy to interpret for a computer. Indeed this is one of the main reasons why computer vision is still in its infancy. On the other hand, inputs that are long strings of '0' and '1' are difficult for humans to interpret, but can be easily used by a computer. After all, computers are designed to manipulate large amounts of information represented as strings of '0' and '1'.

The input we provide must describe the environment without oversimplifying or overcomplicating it. If the input is oversimplified then we risk losing some information about the environment and hence our network will receive an incomplete picture. If the input is overcomplicated then the network will struggle to make generalizations from it and as a result will not learn.

The purpose of this project is not to develop a system that plays exceptionally well in Othello. Instead, we want to develop a system that is able to learn good

strategies starting from scratch, just like a novice human would. Therefore in order to test the agent's learning abilities, we try to avoid providing additional information such as: disc count, mobility and stability.

6.1.1.1 Simple

The simple input representation, as the name suggests, is the simplest way of representing the Othello board without losing any information. The contents of each board square are represented by a single input. Since a white disc is an exact opposite to a black disc it makes sense for them to have opposing input values. Continuing with this logic the value of an empty square must be between that of white and black to avoid any bias. Thus the inputs used were 0.0, 0.5 and 1.0 for white, empty and black respectively. The last input in the representation indicates whose move it is, which is a crucial piece of information for the learning agent. This input is set to 1 if it is black's move and 0 for white's move.

6.1.1.2 Simple2

Unlike the Simple representation, Simple2 uses two inputs for each square. The first input is on if the square contains black, while the second input is on if the square contains white. If both inputs are off then the square is empty (note that both inputs are never on). This representation has both advantages and disadvantages over the Simple representation. Firstly it ensures that all inputs are binary, which is perhaps easier for the network to learn. On the other hand, the amount of information needed to be learned is doubled, which may reduce the convergence rate.

6.1.1.3 Walker

This representation was used by one of the first applications of TD to Othello, and hence was also the one tried for this project (Walker et al. 1994). The input consists of 129 units: two sets of 64 units each and an extra unit indicating the player about

to move. The first 64 units describe the information on the board. This is identical to Simple representation, except without the last input. The second set of 64 units indicates which moves are available to the opponent. In other words, the input is set to 1 if the opponent can place a piece in the corresponding square, otherwise the input is 0. Although the network can deduce whose go it is from the second set of 64 inputs, it is best to provide that information implicitly.

6.1.1.4 Advanced

All the input schemes described previously provide a very basic description of the Othello domain. They rely on the agent to be able to pick up advanced knowledge and strategies. However, due to the complex nature of the domain, some of the more subtle knowledge is very difficult for the agent to recognise. When humans learn to play Othello, they are likely to notice the positional importance of squares (such as corners and x-squares), but without being directly told, they are unlikely to notice the importance of stability and mobility.

In this representation we provide the agent with advanced information such as: number of white and black discs, number of empty squares, current mobility, potential mobility for both players, and stability for both players. The computation of the above information was based on that used by IAGO (Rosenbloom 1982). The following variables were used in the calculation: B = number of moves available to black, W = number of moves available to white, M_1 = number of frontier discs, M_2 = number of distinct empty squares near frontier discs, M_3 = sum of empty squares near frontier discs, S_1 = number of stable non-edge (internal) discs, S_2 = number of stable edge discs. Due to the fact that all other inputs are in the range [0,1], these values had to be scaled to fit that range. In order to do that, the values were divided by the maximum possible value that was determined empirically. Figure 6.1 shows the values used in the implementation.

$$CurrentMobility = \frac{B+1}{B+W+2}$$

$$PotentialMobility = \left\{ \frac{M_1}{27}, \frac{M_2}{37}, \frac{M_3}{84} \right\}$$

$$Stability = \left\{ \frac{S_1}{36}, \frac{S_2}{28} \right\}$$

Figure 6.1 Values used by the Advanced input representation.

It must be noted that this input scheme was developed solely for the purpose of creating a better Othello player, and not for testing the learning algorithms. The extra inputs were not directly learned by the agent and were encoded manually.

6.1.1.5 Partial

In all previous representations the inputs were given as a stream with no distinction made between different rows or columns. The problem with this is that the network does not know the spatial relationships between each square. In other words, it has no idea that the inputs represent a two-dimensional board, as far as it is concerned, the squares are located completely randomly. There must be a way to tell the network that square A1 is adjacent to A2, which in turn is adjacent to A3. Most importantly for Othello, the networks must know that corner squares are adjacent to x-squares.

One method of doing this is to create a partially connected neural network, which has a hidden node for each row, column and diagonal of length 3 or more, so 38 in total. The idea behind this is to provide the network with all combinations of squares that can experience flipping (flips in Othello occur only on straight lines). Figure 6.2 demonstrates this idea for a simple 3 x 3 board.

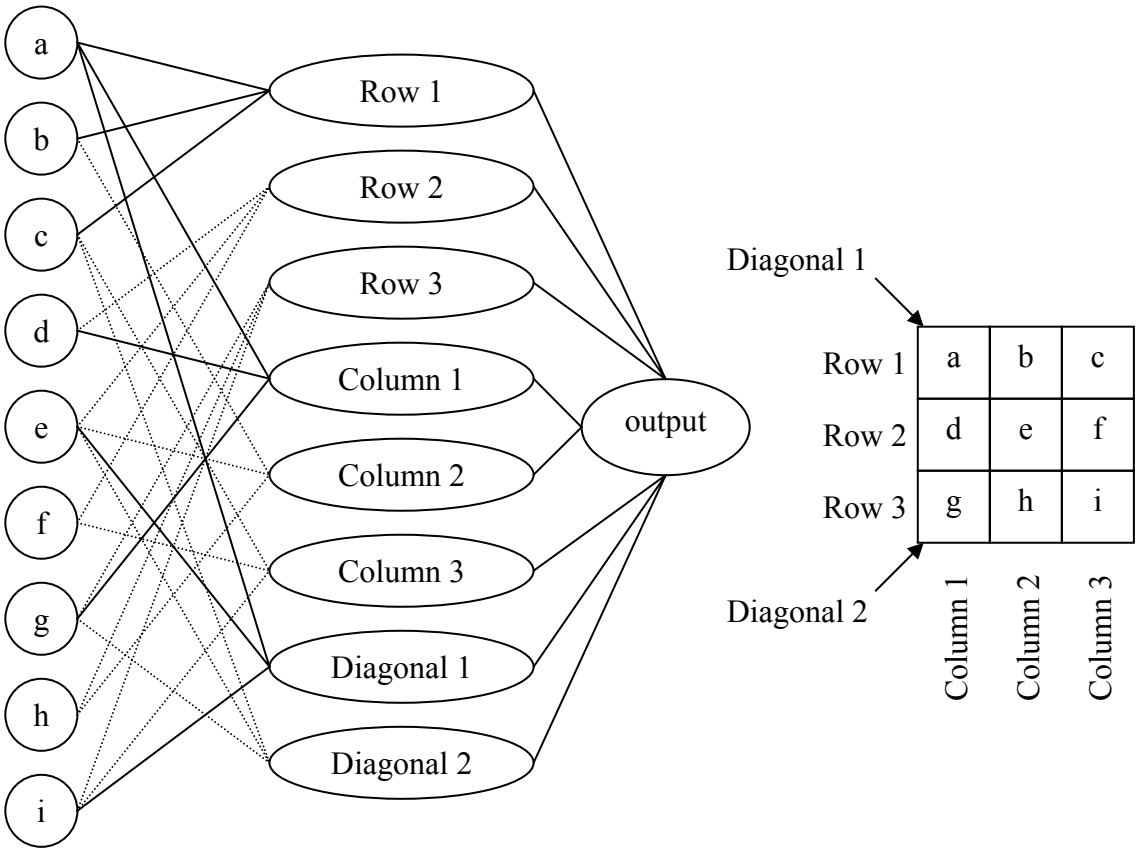


Figure 6.2 An example of a partially connected network for a 3x3 board. For clarity, only connections to Row 1, Column 1 and Diagonal 1 are highlighted.

6.1.2 Action Selection

In this study we train a single network for both players. This means that action selection varies based on whose move it is. The way our reward system is set up, the black player maximizes the reward, while the white player minimizes it. Similarly, the black player must select actions that produce a high output, while the white player selects actions that produce a low output. We have considered 3 strategies for action selection: ε-greedy, softrank and softmax.

6.1.2.1 ε-greedy

ε-greedy action selection was the first selection algorithm used, because it is by far the

easiest to implement. As you recall from section 3.7.1, ε-greedy selects a random action with probability ε and otherwise selects the best available action. The choice of ε was based on the value used by (Vamplew & Ollington 2005a). In the initial experiments ε was set to a constant value of 0.2. Benda suggested that after a period of 15,000 games, exploration should be reduced by changing ε to 0.05 (Benda July 2005, personal communication). All subsequent experiments used this strategy.

Tournavitis proposes a variation to the ε-greedy strategy. The first n moves are played randomly ($4 \leq n \leq 8$) and the remaining moves are chosen with ε-greedy. This strategy was not considered, because it requires the tuning of an additional parameter.

6.1.2.2 Softrank

After using the ε-greedy strategy an attempt was made to implement softmax selection. However, instead of selecting based on the relative values, like in softmax, selection was based on the relative ranking of the actions. In other words, the best ranked action has the highest probability of being selected, followed by the second-best ranked action and so on. This selection method is referred to as softrank. The probability of selecting the i^{th}-ranked action from n available actions is given by:

$$\frac{C-1}{C^i-1} \div \left(\sum_{k=1}^{n} \frac{C-1}{C^k-1} \right)$$

Equation 6.1 Softrank selection.

The constant C used by Equation 6.1 was chosen to be 1.5. For example, if n = 5 then the probabilities are given as {55.3%, 22.1%, 11.6%, 6.8%, 4.2%}. Note that softrank was only used as a trial and is not reported in the results section.

6.1.2.3 Softmax

Recall softmax selection described in section 3.7.2. In theory, this type of selection is better than ε-greedy, because it gives "direction" to the exploration strategy. Softmax

achieves this by increasing the likelihood of selecting actions that are "second-best" and decreasing selection of really bad actions. The temperature parameter τ was initialized to 1 and slowly decayed. The value of τ at the n^{th} episode is given by 0.9999^{n-1}. The constant 0.9999 was used so that τ was close to 0 after 50,000 episodes. This constant is very sensitive to change: lower values prevent exploration, while higher values prevent exploitation. van Eck and van Wezel use the following to determine the value of τ at the n^{th} episode:

$$\tau = \begin{cases} ab^n \ if \ ab^n \geq c \\ 0 \ otherwise \end{cases}$$

Equation 6.2 Determining the value of τ (van Eck & van Wezel 2004).

In Equation 6.2, a = 1, b = 0.9999995, c = 0.02. With such a high value of b, they are performing a much slower decay of τ. At the same time, τ is set to 0 as soon as it goes below the threshold c. However, with the current values of b and c, this occurs only when n is above 7,824,044 – a value which is never reached in my experiments.

6.1.3 Reward

Following Walker's paper, the reward was set to 1 if black won, 0.5 if there was a tie and 0 if white won. There were no intermediate rewards, as all non-terminal states were given 0. With this reward scheme, the network learns the likelihood of the particular player (in our case black) winning from the given state. Output values above 0.5 indicate that black is more likely to win, while those below 0.5 mean that black is more likely to loose, i.e. white is more likely to win.

Some authors such as (Leouski 1995) use a reward that is based on the final disc difference between the network and its opponent. As a result the network learns the importance of winning by a larger margin and loosing by a smaller margin. However in trial runs, this scheme failed to produce good results, and hence was ignored.

6.1.4 Parameters common to all network architectures

Before training could begin, a number of parameters had to be established by running multiple single-trial experiments for each set of parameters. Once the parameters have been established, 10 trials of networks were trained and tested for a particular set of optimal parameters. Note that due to time constraints it was not feasible to run 10 trials for every set of parameters.

6.1.4.1 Training length

Walker et al. used a training length of just 30,000 episodes, a number that was probably influenced by the limitations of the hardware of that day (Walker et al. 1994). Nevertheless, 30,000 episodes seems reasonable and just to be sure a training length of 50,000 episodes was chosen. Longer training length was not seriously considered for two reasons. The first and the major reason is the limited availability of time and computing resources. Finally, results obtained by Cranney indicate that there is very little variation for networks trained between 80,000 and 2,000,000 episodes (Cranney 2002). My trial experiments that went for 500,000 episodes indicate a similar result. Finally, because we are comparing two types of neural networks, it was important to keep the training length the same for both types of networks.

6.1.4.2 Discount

Discount factor γ was set to 1 for all experiments. This is because we only care about winning and not about winning as fast as possible.

6.1.4.3 Lambda

The lambda parameter λ controls the number of steps in the TD backup. Various lambda values have been tested: 0, 0.3, 0.5, 0.7, 0.8, 0.9 and 1. Many authors have found that changes in lambda produce very little variations in the final outcome

(Deshwal & Kasera 2003; Tournavitis 2003; Walker et al. 1994). As an exception to this rule, some authors have found that setting λ to 1 gives high fluctuations in weights and produces the worst result (Deshwal & Kasera 2003; Tournavitis 2003). Both of these claims are supported by this study. For all experiments λ was set to 0.7, which was also the value used by Tesauro in TD-Gammon (Tesauro 1994).

6.1.4.4 Eligibility traces

For most experiments the accumulating trace was used. The replacing trace was also tried, but it did not produce any improvement in results.

6.1.4.5 Weight initialization

Weight initialization is important for neural networks. If all weights are initialized to the same value then they will get updated at the same rate and will not be able to learn. Therefore, it is common to initialize weights to random values within the range [-β, β]. The value of β used for all experiments was 0.5.

6.1.5 Fixed architecture network

The fixed architecture network used in the experiments was an MLP as described in section 4.6. The network had a single hidden layer and one output node in the output layer. For all experiments the hidden nodes use a symmetric sigmoid function. In the initial experiments, the output node used a linear activation function. Later it was decided to change this to an asymmetric sigmoid function. The reason for this is the fact that the sigmoidal function can keep the output in the desired range of [0, 1].

6.1.5.1 Training algorithms

The MLP networks were trained using the Sarsa(λ) and Q-Learning(λ) algorithms

outlined in Figure 6.4 and Figure 6.5. The Q-Learning(λ) algorithm differs from Sarsa(λ) by training on the action that was selected greedily, while executing an action that was selected non-greedily. Both algorithms use the SELECT method which is shown in Figure 6.3. The SELECT method is based on ε-greedy action selection. The method takes in the following parameters: a random value in the range [0, 1], a set of available moves, state s' and the value of that state. If the random parameter is set to a constant value of 1, the method selects an action greedily, as used by Q-Learning(λ) in Figure 6.5.

```
If random ≤ ε
        s' ← s with any move from availableMoves executed
        Qt ← network output for state s'
Otherwise
        If currentPlayer == black then bestValue ← -infinity
        Otherwise bestValue ← +infinity

        For each move m in availableMoves
                s2 ← s with move m executed
                value ← network output for state s2
                If (currentPlayer == black and value > bestValue) or
                (currentPlayer == white and value < bestValue)
                        bestValue ← value
                        s' ← s2

        Qt ← bestValue
```

Figure 6.3 SELECT method. Implements ε-greedy action selection.

```
For each learning episode
        Clear all eligibility traces
        For each step of episode
                availableMoves ← possible moves from current state s
                If |availableMoves| == 0 (current player passes)
                        s' ← s with opponent player
                        Q_t ← network output for state s'
                Otherwise
                        SELECT(random, availableMoves, s', Q_t)

                If this step is not the first in the episode
                        δ_TD ← r + (Q_t – Q_t-1)
                        Update all weights to minimize δ_TD
                        Recalculate network activations
                Q_t-1 ← Q_t
                Update eligibility traces for all network weights
                Execute s' and observe reward r
        Perform one more learning pass to learn about the final action
```

Figure 6.4 Sarsa(λ) algorithm for training MLPs.

```
For each learning episode
        Clear all eligibility traces
        For each step of episode
                availableMoves ← possible moves from current state s
                If |availableMoves| == 0 (current player passes)
                        s' ← s with opponent player
                        G ← network output for state s'
                Otherwise
                        SELECT(1, availableMoves, s', G)

                If this step is not the first in the episode
                        δ_TD ← r + (G – Q_t-1)
                        Update all weights to minimize δ_TD
                        Recalculate network activations

                SELECT(random, availableMoves, s', Q_t)
                Q_t-1 ← Q_t
                Update eligibility traces for all network weights
                If Q_t ≠ G clear all eligibility traces
                Execute s' and observe reward r
        Perform one more learning pass to learn about the final action
```

Figure 6.5 Q-Learning(λ) algorithm for training MLPs.

6.1.5.2 Learning rate

The learning rate α controls how much the weights get modified during training. High learning rates tend to give faster convergence rates, but are also susceptible to overshooting local minima. Three learning rates have been tried for the MLP: 0.01, 0.05 and 0.1. For most input representations the learning rate made little difference to the final result. For the Walker input scheme however, higher learning rates produced results that were significantly better.

6.1.5.3 Number of hidden nodes

Potentially, each hidden node can become a feature detector. So the number of hidden nodes controls the network's ability to develop feature detectors. The numbers tested were: 1, 5, 8, 30 and 50. Most of the experiments used 30 hidden nodes as recommended by (Walker et al. 1994). Numbers above 50 were not considered, because it seems unreasonable to have more hidden nodes than the number of inputs. After the success of the network with just 1 hidden node, an attempt was made to completely eliminate the hidden layer. However this proved to be too difficult to implement as it requires the removal of the backpropagation algorithm.

6.1.6 Constructive architecture network

The Cascor network (Fahlman & Lebiere 1990) was chosen as the representative of the constructive architecture. Just like in MLPs, the hidden nodes in the Cascor networks use a symmetric sigmoid function and the single output a linear function.

The original Cascor algorithm was not intended to be used for Reinforcement Learning. The authors of (Bellemare et al. 2004; Rivest & Precup 2003) modify the Reinforcement Learning process so that Cascor can be applied. The modification involves two alternating stages. In the first stage, the agent selects and executes

actions, and then caches the input state with the target value generated via TD. Once the cache is full, the network is trained on the cached examples using the Cascor algorithm. After training, the cache is cleared and the algorithm returns to the first stage. This new algorithm performs well on Tic-Tac-Toe and car-rental, but does not do so well on the more complex Backgammon task.

In our experiments we modify the Cascor process so that it can be directly incorporated into existing Reinforcement Learning algorithms. The new algorithm is based on Cascade2, developed by Fahlman as reported in (Prechelt 1997). Cascade2 differs from Cascor by training candidates to directly minimise the residual error rather than to maximise their correlation with that error. This is achieved by training the output weights, as well as the input weights of each candidate. When the candidate is added to the network, both its output and input weights are transferred.

6.1.6.1 Weight update algorithm

The original Cascor algorithm uses Quickprop to update the weights (Fahlman 1988). Quickprop uses an estimate of the second derivative of the error with respect to the weights to produce faster training of the network. However, Quickprop was designed for batch-training applications, and hence cannot be used in our situation, where the weights are updated after every forward pass of the network. For our experiments, the standard backpropagation algorithm was used instead.

6.1.6.2 Training algorithms

For most experiments, the cascade networks were trained using the modified version of the Cascade-Sarsa algorithm, which is described in (Vamplew & Ollington 2005a). This algorithm (Figure 6.6) differs from Cascade-Sarsa, because it is learning state values and not state-action pairs. For some experiments, the cascade networks were trained using Q-Learning, which is a combination of Figure 6.5 and Figure 6.6.

```
P_current ← 0, P_last ← infinity
For each learning episode
        Clear all eligibility traces
        For each step of episode
                availableMoves ← possible moves from current state s
                If |availableMoves| == 0 (current player passes)
                        s' ← s with opponent player
                        Q_t ← network output for state s'
                Otherwise
                        SELECT(random, availableMoves, s', Q_t)

                If this step is not the first in the episode
                        δ_TD ← r + (Q_t – Q_t-1)
                        Update only output weights to minimize δ_TD
                        Update candidate weights
                        Recalculate network activations

                Q_t-1 ← Q_t
                Update eligibility traces for all network weights
                Execute s' and observe reward r
                P_current ← P_current + (δ_TD)^2
        Perform one more learning pass to learn about the final action

        Once every P episodes, where P = patience length
                If P_current ≥ P_last * patienceThreshold
                        Add candidate with the lowest error
                        P_last ← infinity
                Otherwise P_last ← P_current
                P_current ← 0
```

Figure 6.6 Modified version of Cascade-Sarsa (Vamplew & Ollington 2005b).

6.1.6.3 Learning rate

A number of learning rates have been tried for Cascor networks: 0.001, 0.005, 0.01, and 0.1. The only learning rate that gave consistently good results was 0.001. Anything above that value either gave poor results or took an incredibly long time to train. On further analysis it was established that Cascor using learning rate 0.1 had weights that diverged to NaN. This divergence in weights suggested that the output was exceeding the normal [0, 1] range. When the output was changed to use a sigmoidal function this problem was solved, and the networks could be trained with higher learning rates.

6.1.6.4 Number of candidates

The number of candidates is the number of units used to select a new hidden node. This number was always set to 8 as used by (Vamplew & Ollington 2005b).

6.1.6.5 Maximum number of hidden nodes

This controls the maximum number of hidden nodes that can be added to the network. As the MLPs used 30 hidden nodes, this number was also set to 30.

6.1.6.6 Patience threshold

The patience threshold controls how much the error is allowed to change before a new hidden node is added. If currentError ≥ previousError x patienceThreshold then we add a new hidden node. The patience threshold was always set to 0.95 as used by (Vamplew & Ollington 2005b).

6.1.6.7 Patience length

The patience length is the minimum number of episodes that need to occur before a new hidden node can be added. This number was adjusted such that the network reaches the maximum number of hidden nodes before training is complete. For a period of about 2,000 episodes the networks do not add any hidden nodes. After that they add hidden nodes in a linear fashion, averaging about 833 episodes per hidden node (~2.5 patience lengths). The networks reach their maximum capacity of 30 hidden nodes at about 27,000 episodes:

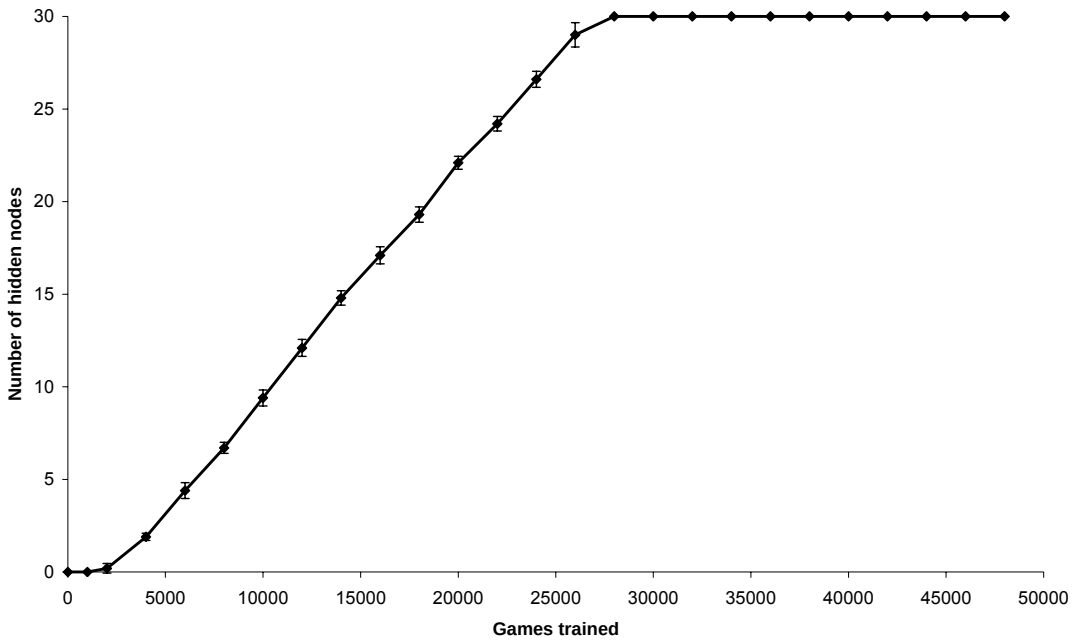


Figure 6.7 Rate of addition of hidden nodes for Cascor (10 trials).

6.2 Testing environment

The testing environment (OthelloDK) was used as a means to test the quality of the trained neural networks against a range of in-built opponents. OthelloDK uses a graphical representation of the Othello board (Figure 6.8). In the two drop-down boxes the user can select who will play with black and white discs respectively. The "Number of games" field is used to set the number of games that will be played. If the field is set to 1 then the user will be shown a graphical representation of the game, where each move is controlled with a click of a mouse button. Alternatively, if the field is set to N (N > 1) and both players are not "Human" then OthelloDK will play N number of games (without graphical output) and display the result in the command line. This was useful when a large number of games needed to be played quickly. The output included information about who was playing, total number of black discs, total number of white discs, number of wins by black, number of wins by white, number of ties, total number of games played and the time taken to play those games.

During the game, OthelloDK displays the set of possible moves for the current player

(little red squares in Figure 6.8). By pressing the evaluate buttons it is possible to find the evaluation for those moves given by the neural network; “Evaluate Left” for neural network playing with black and “Evaluate Right” for the net playing with white. For each possible move the evaluation score is shown in the corresponding square. The best score for the current player is highlighted in yellow.

OthelloDK can also support custom-built board configurations. Discs can be added and removed by clicking the right mouse button. This allows the user to create custom scenarios or reproduce previous games.

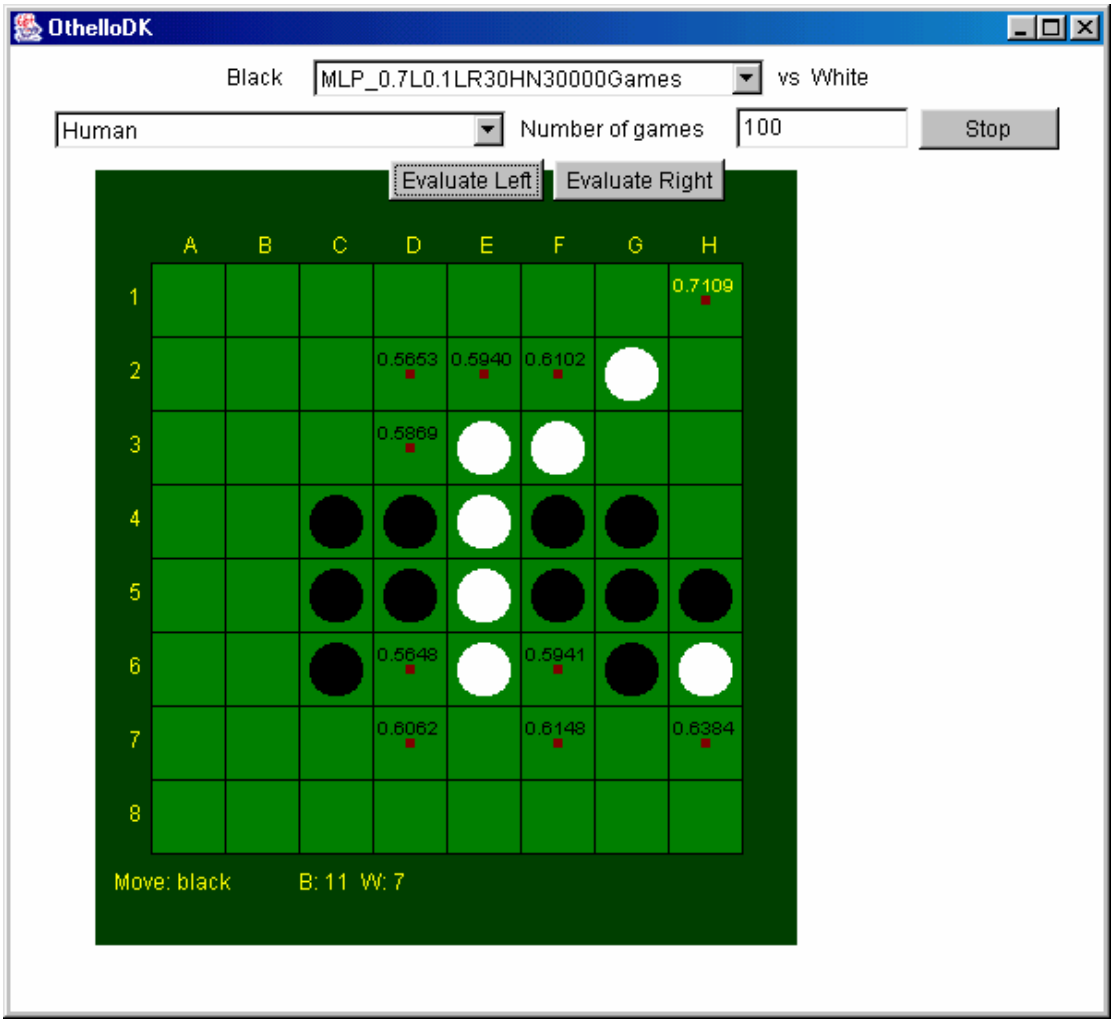


Figure 6.8 OthelloDK testing environment.

6.2.1 Built-in Opponents

The primary opponent used for testing was the Random player. The Random player randomly selects a move from the set of available moves. If our neural network has learned any strategies then we can expect it to play better than the Random player, i.e. it should win more than 50% of the time. If the network wins significantly less than the Random player then we know that it has learned the strategy for the opposite player.

It is possible that the neural network learns how to consistently beat the random player, but learns nothing else. To avoid such situations the Greedy player was introduced. The Greedy player (described in section 5.3.1) is not substantially better than the Random player (Table 6.1), but does add some variety to the testing process. In brief, the Greedy player chooses an action that maximizes his disc count. If there is more than one such action then the Greedy player chooses one of those actions randomly. It must be noted that such greedy action selection makes the Greedy player not so good for testing purposes, as he is likely to choose the same action for a given situation. To accommodate this, ε-greedy action selection with ε = 0.1 was used for the neural networks. As a result the networks did not play at their optimal level (Table 6.2), although they were close to it. Perhaps it would be better to play neural networks greedily and use ε-greedy selection for the Greedy player instead. Unfortunately such an approach was not considered at the time of testing.

% Wins, Ties, Losses		**2nd player**		
		Random	**Greedy**	**Random2**
1st player	**Random**	44.6, 4.1, 51.3	37.9, 3.4, 58.7	20.2, 3.3, 76.5
	Greedy	59.8, 3.2, 37.0	50.1, 3.6, 46.3	31.4, 3.4, 65.2
	Random2	71.8, 3.5, 24.7	65.2, 3.6, 31.2	45.1, 4.2, 50.7

Table 6.1 Percentage of wins, ties and losses for the first player.

	% Wins	**% Ties**	**% Losses**
ε = 0.1	87.5%	2.4%	10.2%
ε = 0.0	89.3%	2.3%	8.4%

Table 6.2 Effect of ε (during testing) on MLP's performance against Random player.

Taking corners when they are available is an important part of winning in Othello. Both Random and Greedy fail to do this and hence it was decided to introduce another player – Random2. If there is a corner available then Random2 will take it, otherwise it will choose a move randomly, just like the Random player. Random2 is considerably better than both Random and Greedy players (Table 6.1).

6.2.2 Measuring learning

It is important to be able to evaluate the quality of the learned policy. Usually in Reinforcement Learning problems, learning can be measured directly as the total amount of received reward. However, this is not the case in a gaming context where agents are trained through self-learning, because in these situations the accumulated reward remains approximately constant. In such situations, the policy of the agent can be evaluated by making it play against opponents whose abilities are known. Sometimes the agent learns how to beat just one type of opponent and hence it is important to test the agent against a range of opponents that have varying strategies. The quality of a player can be measured as the likelihood of him winning against his opponent. Since ties are possible in Othello they must also be taken into account. So a better measure for a player's quality would be the likelihood of him not losing against his opponent.

Some authors (Leouski 1995; Moriarty & Miikkulainen 1995) have adopted the following strategy for testing. Generate all the combinations of the first 4 legal moves from the initial board configuration. Play a game from each of the 244 resulting board states (60 unique states if we ignore rotations and reflections). However, it seems that playing 244 games is not enough for meaningful statistical data. To prove this hypothesis, 100 rounds of 244 games each were played between two Random players. As the result of this experiment, the first player won 45.3% with a standard deviation of 3.2%, while the second player won 50.5% with a standard deviation of 3.5%. Based on the averages, we know that in a sample of many games the second player will win more often than the first player. However in a sample of just 244 games, statistically there is a 15.8% chance that the first player will win 48.5% or more, and a

15.8% chance that the second player will win 47.0% or less. We suddenly find that nearly 1 in 6 rounds can go the opposite way (with first player winning more).

Hence it is important to keep the number of games played large, but obviously not too large as testing would take too long. Those experiments that were based on just one trial were tested on 10,000 games. When an experiment used 10 trials, each of the 10 networks was tested for a 1,000 games. The result of each trial was averaged and its 95% confidence interval determined using the following:

$$X_{av} \pm 1.96\left(\frac{\sigma}{\sqrt{n}}\right)$$

Equation 6.3 Confidence interval.

In Equation 6.3 X_{av} is the average value of trials, σ is the standard deviation and n is the number of trials. From Table 6.1 we can see that the Random player going first wins just 44.6%, but it wins 51.3% when going second against another Random player. The same discrepancy is found when Greedy plays itself and when Random2 plays itself. This indicates that the starting order can have a significant impact on the final outcome. Therefore for single-trial experiments neural networks were tested playing with both black (first player) and white discs (second player), and the result was averaged.

6.2.3 Other features

The testing environment also has the ability to save games. The saved games can be loaded with WZebra (Andersson & Ivansson 2004). WZebra is a sophisticated Othello program that is capable of looking up to 24 moves ahead, and can provide a move-by-move game analysis such as the one shown in Figure 6.9. WZebra can also provide evaluations of each available move similar to that shown in Figure 6.8. This analysis is useful when measuring the quality of the neural networks.

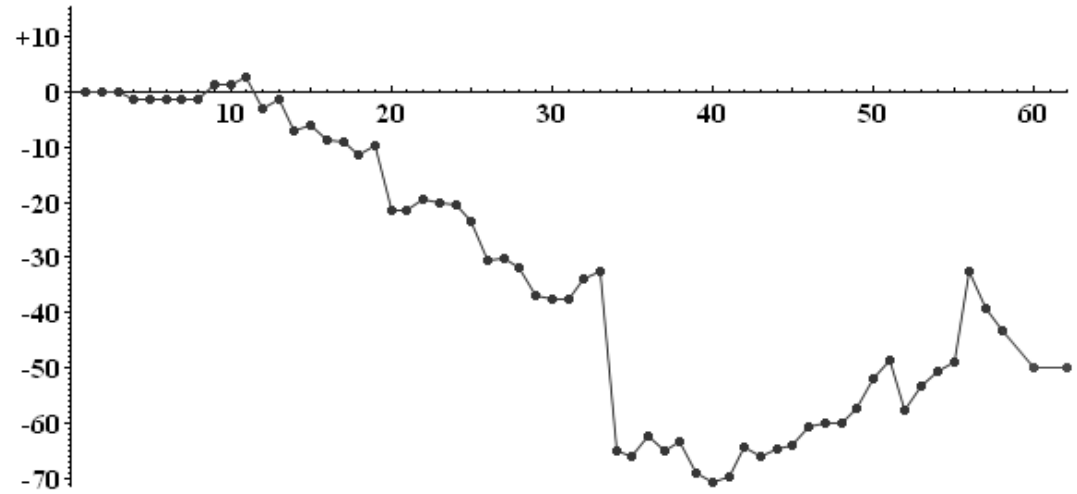


Figure 6.9 Game analysis produced by WZebra. Positive scores are good for black (Andersson & Ivansson 2004).

Every 1,000 training episodes, the information related to each neural network is saved into a file. This file contains information such as the number of inputs, number of hidden nodes and their weights. Assuming we are using an MLP with Simple input, it is possible to construct a weights map (see Figure 7.10 in section 7.2.2) that highlights the relevant importance of each input. The RGB value of each square i is given as (W_iF, W_iF, W_iF). F is a multiplicative factor between 20 and 100. W_i is calculated based on the input weights to hidden nodes h (w_{ih}) and output weights from hidden nodes h (o_h):

$$W_i \leftarrow \sum_h w_{ih} o_h$$

Equation 6.4 W_i calculation.

Before W_i can be used, it must be normalized:

$$W_i \leftarrow W_i - \text{MIN}\{W_0, W_1, \ldots, W_{63}\}$$

Equation 6.5 W_i normalization.

The resulting colour of each square is a shade of grey (later recoloured to blue), with dark colours corresponding to low weights and light colours corresponding to high weights.

Chapter 7 Results and Discussion

7.1 MLP vs Cascade

In this section we compare the performance of MLP networks against Cascade networks. To avoid any bias, both types of networks were trained with identical parameters: Walker input scheme, softmax action selection, 30 hidden nodes, 50,000 training episodes and $\lambda = 0.7$. The only difference was in the learning rate, with Cascade using 0.001 and MLP using 0.01. 10 trials of each type of network were trained and tested against the Random player at intervals of 2,000 games (Figure 7.1). The result for the first point (after 0 games trained) was obtained by playing Random against Random; this is based on the assumption that the networks play completely randomly at the start of training.

After the first 1,000 training episodes both networks improve equally to 60%, but with a huge variation of ± 5%. After that, the performance of Cascade accelerates rapidly, reaching a near-maximum of 80% after just 8,000 training episodes. In contrast, the MLP network learns much slower, taking about 26,000 training episodes to reach the same performance. After 26,000 training episodes the confidence intervals of the networks overlap and we cannot conclude that one network is better than the other. Due to its structure, Cascade uses less hidden nodes during the initial training period. With fewer weights to adjust, Cascade is able to converge much faster than MLP. One might wonder how the Cascade manages to reach good performance with so few hidden nodes. In section 7.4 we show that the number of hidden nodes does not affect performance.

As we increase the learning rate, the performance of Cascade stays the same, while the performance of MLP improves (Figure 7.2). From Figure 7.2, we can see that Cascade converges even faster than before (Figure 7.1), but its performance is poor and very unstable, as indicated by long error bars. The performance of MLP on the other hand is very consistent.

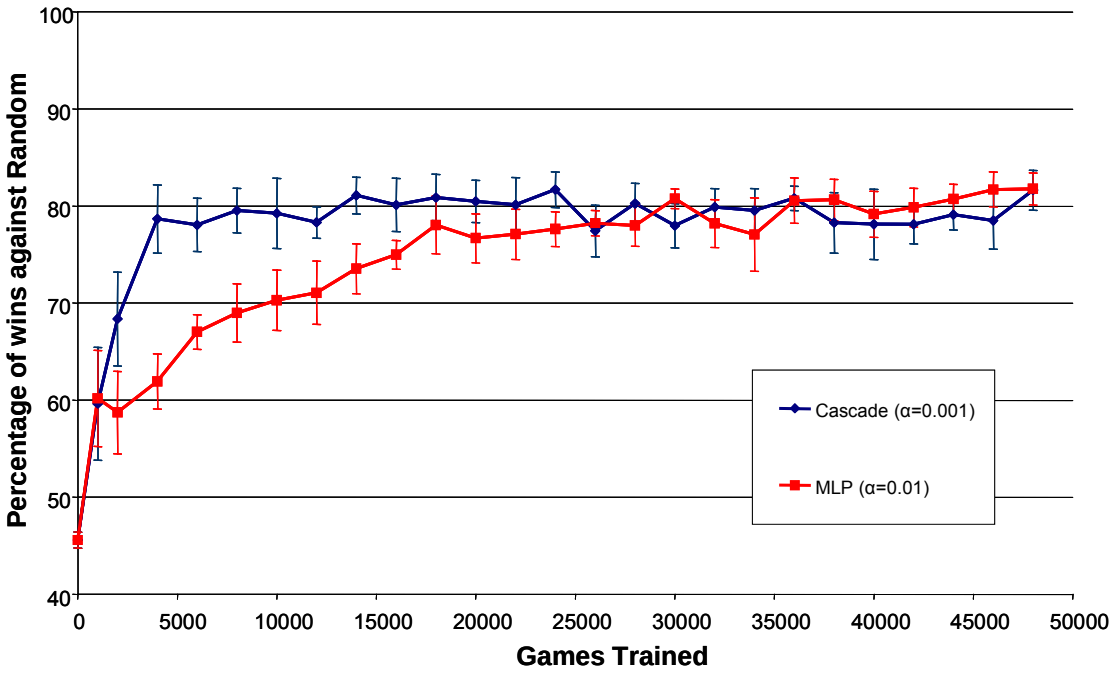


Figure 7.1 MLP and Cascade performance for low learning rates (10 trials).

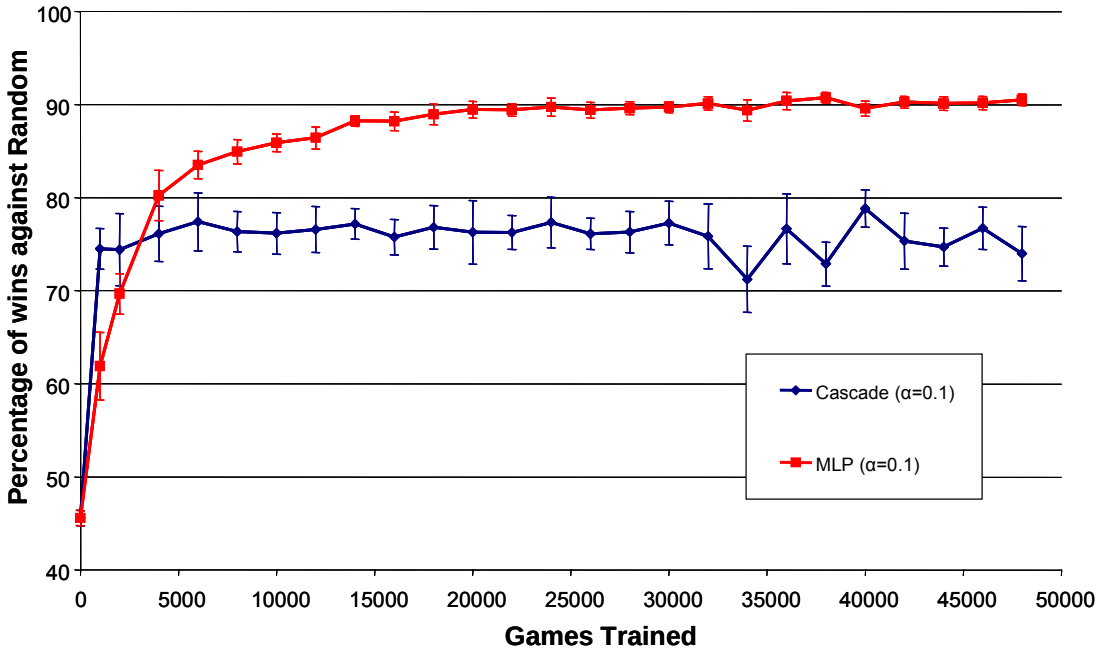


Figure 7.2 MLP and Cascade performance for high learning rates (10 trials).

7.2 What did the networks learn?

In this section we analyse the networks' learning ability. In particular, we show which concepts the MLPs with Walker input have learnt and those that have eluded them. To achieve this we use three methods: move evaluation, weights map and playing against in-built opponents.

In move evaluation, the network is presented with a particular board, designed to test its strengths and weaknesses. The network evaluates each available move, displaying high values for good moves and low values for bad moves. The weights map technique looks at the learned values for each square and is used to check the static evaluation of the whole board. Finally, we play the networks against in-built opponents to test their ability to generalize to different playing strategies.

7.2.1 Move evaluation

Consider Figure 7.3. The networks evaluate this position as highly winning for white. We know from Figure 5.3 in section 5.3.1 that this is not true. The reason for this is that the networks are trained with no look-ahead and hence they evaluate the board statically. In this case, the white is given a high evaluation because he is controlling important squares such as a-squares and b-squares. We also know that similarly-matched opponents, like those used during self-play, are unlikely to experience such a board position. So the networks are unlikely to evaluate it correctly.

The board position in Figure 7.4 was used by Cranney to determine whether the networks have learned the mobility strategy (Cranney 2002). The best move for black in this situation is E8, because then the white is left with just two moves (B2 and G2) that both give away corners. Instead the network chooses H4.

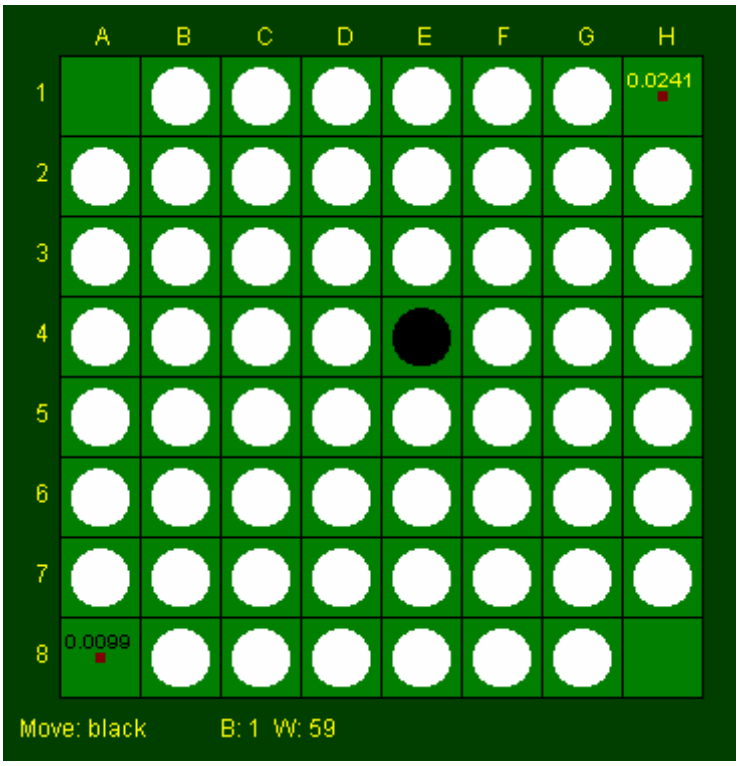


Figure 7.3 Learning to predict the future outcome.

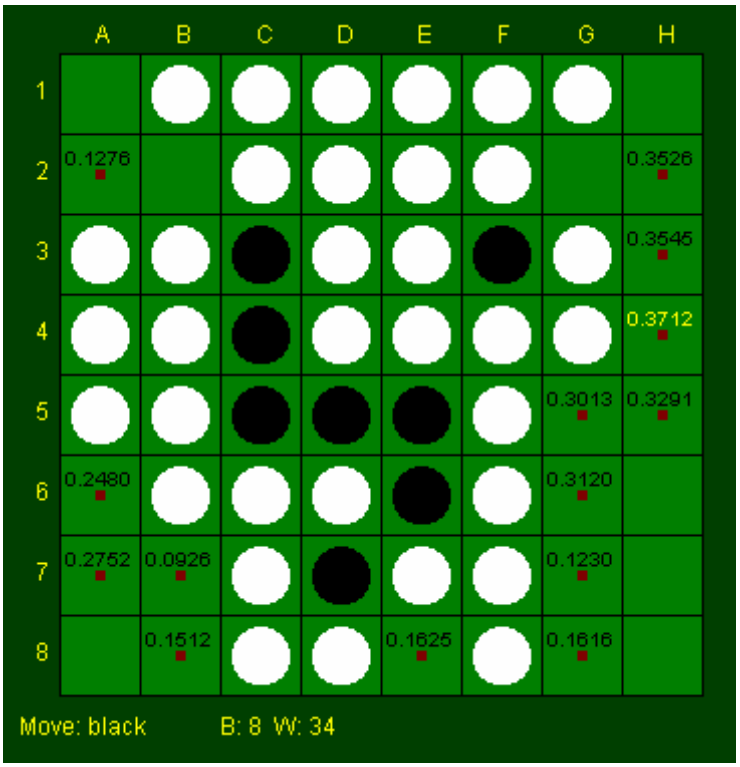


Figure 7.4 Learning mobility.

This choice is based more on the positional strategy, because it captures a b-square on an empty edge. In fact the top 4 moves favoured by the network are on the empty edge. We can also see that the network has learned not to give away corners as those moves (A2, B7, B8, G7 and G8) are ranked the lowest.

The boards in Figure 7.5 test whether the network has learned to survive. If black chooses one of the following moves: C2, C4, C6, G2, G4 and G6; then he will be completely wiped out in the next turn. Clearly the networks have not learnt to survive when faced with the board in Figure 7.5 Left. However, the networks can survive if survival involves taking an edge square as shown in Figure 7.5 Right.

Consider Figure 7.6. Out of a possible selection of 9 moves the network favours the one that eliminates the opponent (Figure 7.6 Left), so it has learnt to win the game when it can do so. However when given the opportunity to capture a corner, the network "forgets" about winning and goes for the corner (Figure 7.6 Right). This occurs because corners have a major precedence over any other location. Perhaps it is beneficial to train the networks with a discount factor, because then they will attempt to win the game as soon as possible.

Figure 7.7 shows that the networks have learnt to protect corners. From a choice of 21 moves the networks choose the only move (A5) that will stop white from taking the corners. In fact, the networks value this move 30% more than any other move. In contrast, Figure 7.8 shows that the networks fail to recognize the importance of corner-seizing moves. In Figure 7.8 Left, black can take the bottom left corner by going to G8. In Figure 7.8 Right, black can take the bottom left corner by going to E7.

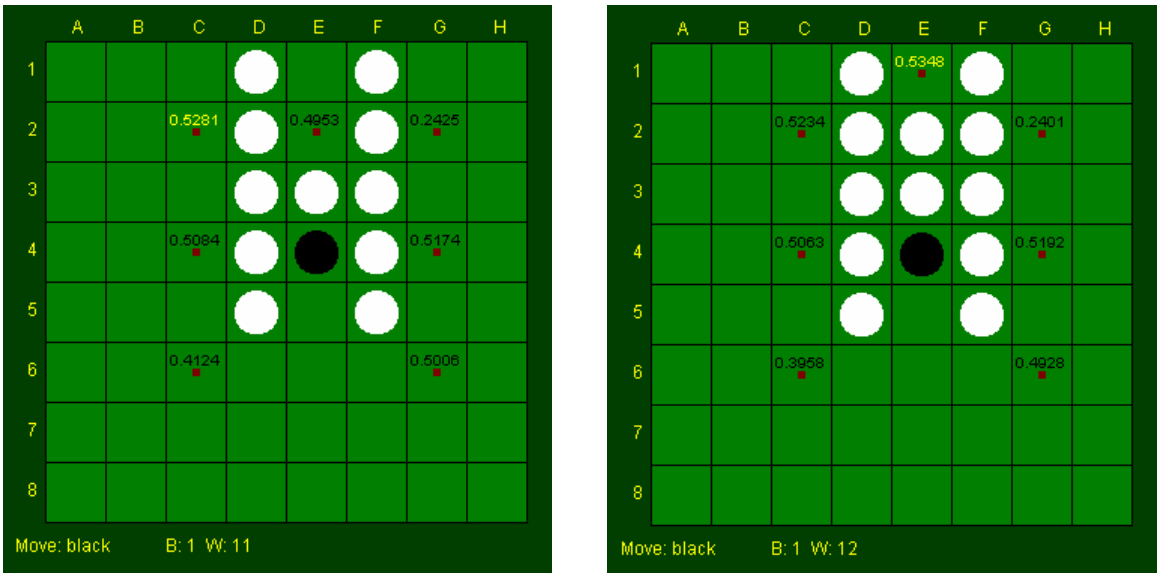


Figure 7.5 Learning to survive.

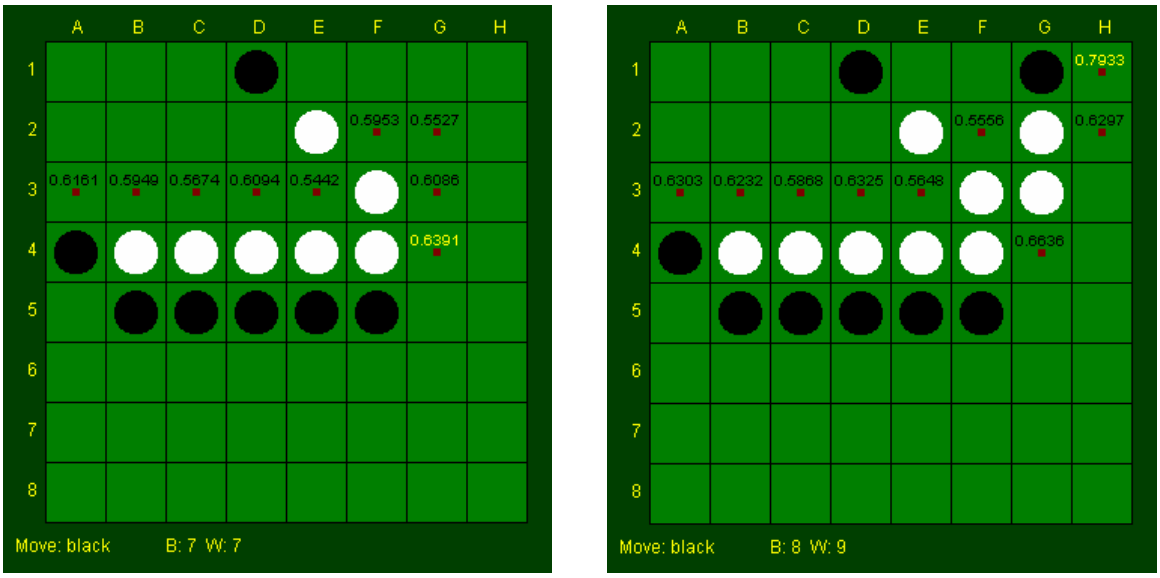


Figure 7.6 Learning to eliminate the opponent.

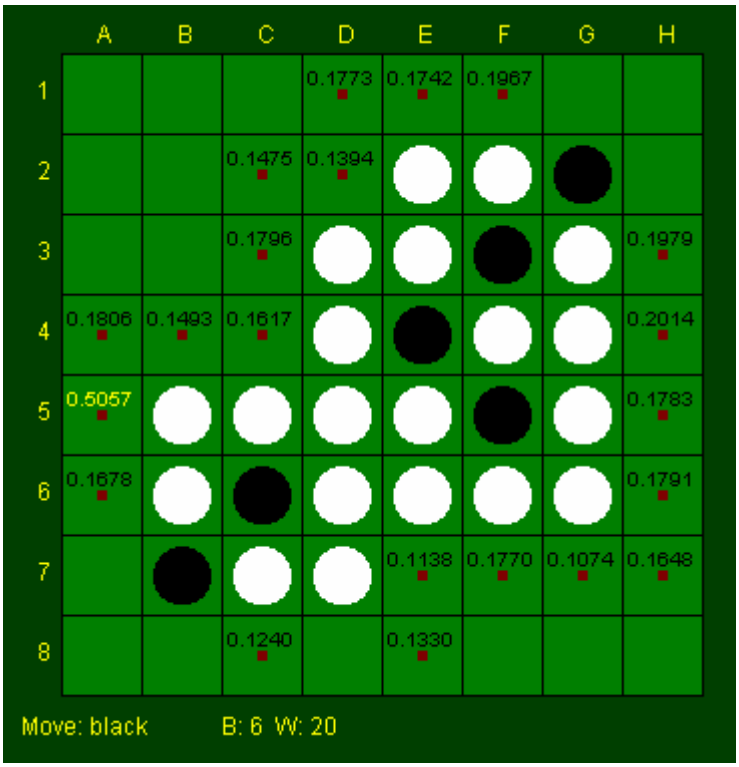


Figure 7.7 Learning to protect corners.

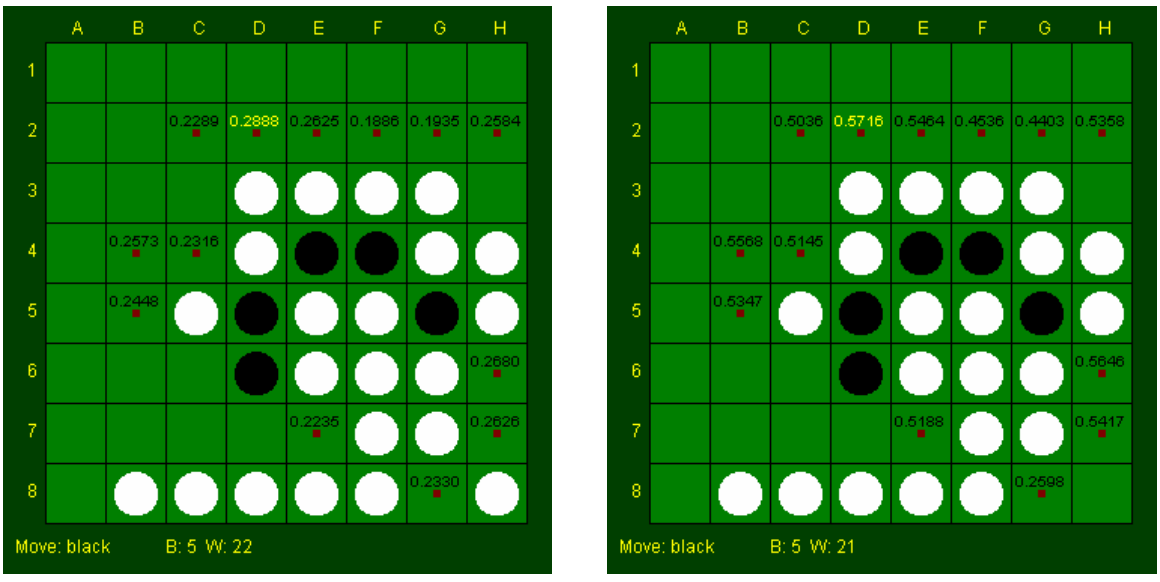


Figure 7.8 Learning to recognize corner-seizing moves.

7.2.2 Weights map

van Eck and van Wezel use a position player that evaluates each position based on the values given in Figure 7.9 Left. The values of the corners are highest (100), while the values of the x-squares are lowest (-50), and thus they are the best and worst positional locations. x-squares and c-squares have low values because they can potentially give away corners, with x-squares more likely to do so. Furthermore, these squares make it impossible for the player to obtain the adjacent corner from that direction for the rest of the game (van Eck & van Wezel 2004). a-squares (10) and b-squares (5) have positive values, because they are semi-stable and are good for controlling other locations. The inner squares (-1) and outer squares (-2) are almost neutral for the positional player as they are neither good nor bad. Figure 7.9 Right is a graphical representation of Figure 7.9 Left, with positive locations shown in darker shades and negative locations shown in lighter shades.

Figure 7.10 shows a weights map for an MLP with one hidden node. We can assume that the darker the square the more valuable it is for the agent. It must be noted however, that these maps only provide a rough guide to the network's learned policy. In reality, we have little idea about how the input weights interact with each other or whether a higher weight carries greater significance than a lower one.

Each location in Figure 7.10 has been ranked from 1 to 64, where 1 is the darkest square and 64 is the lightest square. We can see a great deal of similarity between this map and the 'ideal' map shown in Figure 7.9 Right. The biggest standout is the corners, which are much darker than any other location. The x-squares and c-squares are relative light, as they should be. Furthermore, we can see that some of the edge squares are darker than the outer and inner squares. Based on the low differences in contrast, it may seem that the network would struggle to differentiate the squares. However, we must remember that the network is mostly concerned with the ranking of a particular square and not its actual value. Based on Figure 7.10, we can calculate the average ranking of each type of square: corner – 2.5, a-square – 9.625, b-square – 18.25, outer – 36.75, inner – 39, c-square – 50.125 and x-square – 58.5.

From the above, we can see that corners are ranked highly, while x-squares are ranked

very low, closely followed by c-squares. The outer and inner squares are very similarly ranked, although inner squares are ranked lower, which slightly contradicts Figure 7.9 Left.

	A	B	C	D	E	F	G	H
1	100	-20	10	5	5	10	-20	100
2	-20	-50	-2	-2	-2	-2	-50	-20
3	10	-2	-1	-1	-1	-1	-2	10
4	5	-2	-1	-1	-1	-1	-2	5
5	5	-2	-1	-1	-1	-1	-2	5
6	10	-2	-1	-1	-1	-1	-2	10
7	-20	-50	-2	-2	-2	-2	-50	20
8	100	-20	10	5	5	10	-20	100

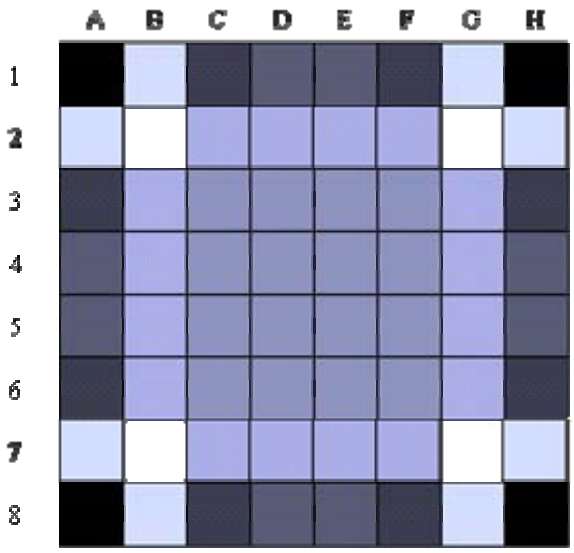


Figure 7.9 Positional strategy. Left: square values based on (van Eck & van Wezel 2004). Right: values on the left converted to a weights map.

1	35	8	14	13	5	51	2
53	64	37	34	40	24	57	54
10	59	33	44	50	32	23	7
12	55	25	38	49	41	45	11
29	58	22	43	60	48	21	16
9	46	47	30	36	26	18	6
62	52	42	28	39	19	61	27
4	63	17	20	31	15	56	3

Figure 7.10 Weights map learned by MLP with Advanced input (1 trial).

7.2.3 Playing against in-built opponents

Another method of testing the networks is to play them against in-built opponents. Figure 7.11 shows the performance of an MLP that used Simple input representation. Even with such a basic input description, the network was able to beat Random 85% of the time. This result is also supported by Cranney's work, whose networks achieved around 80% when playing black (Cranney 2002). This shows that the network has definitely learnt something, as otherwise it would win around 50% against Random.

Due to the nature of Othello, the Random player is not so bad during the opening stages of the game where it is important to keep discs evenly distributed in inner squares. However, later in the game the Random player is likely to make two crucial mistakes: he does not take corners when given the opportunity and gives away corners by going to x-squares and c-squares. These mistakes are enough to tip the balance of an otherwise even game. Even if one plays perfectly in the opening and then switches to Random, he will still lose every time to the most basic opponent.

Surprisingly, the network's performance against Greedy was not much worse than against Random. Again this result can be reduced to the fact that Greedy does not try to capture corners, and simply goes for the move that flips more discs. Nevertheless, the Greedy strategy is considerably different to Random, meaning that the networks were able to play against different strategies.

From the discussion so far, it may seem that the only reason the networks are winning is because they have learnt to capture corners. But is this all they have learnt? To answer this question, we test the networks against Random2. Although, the network's performance has dropped to 68%, they are still winning more than losing (Figure 7.11). This means that they have also learnt the importance of other squares that lead to the capture of corners. When Random2 was modified so that it also avoids x-squares, the performance of networks did not drop significantly.

Although the networks received a reward of 0.5 for a tie, Figure 7.11 shows that the number of ties is low (<3%). So the networks have indeed learnt to win, rather than tie every game. It is also the case that ties occur very rarely, both in training and

testing, and hence the rarely-occurring reward of 0.5 becomes negligible. We can find further statistics about the games played:

	average margin	average number of remaining squares
vs Random	20.64	2.10
vs Greedy	17.90	0.83
vs Random2	11.05	1.86

Table 7.1 Statistics for MLP against in-built opponents (1 trial).

As the number of wins declines from Random to Random2, so does the average winning margin (Table 7.1). The second column in Table 7.1 describes the average number of empty discs remaining at the end of each game. A standard game of Othello finishes with all 64 squares taken. Occasionally games finish prematurely, when there is a wipe-out of one player or when neither player can make a move. It is interesting to note that there was a lower number of remaining squares when the network played against Greedy. Perhaps, it is much harder to achieve a wipe-out against a Greedy player, because he is always maximizing his number of discs.

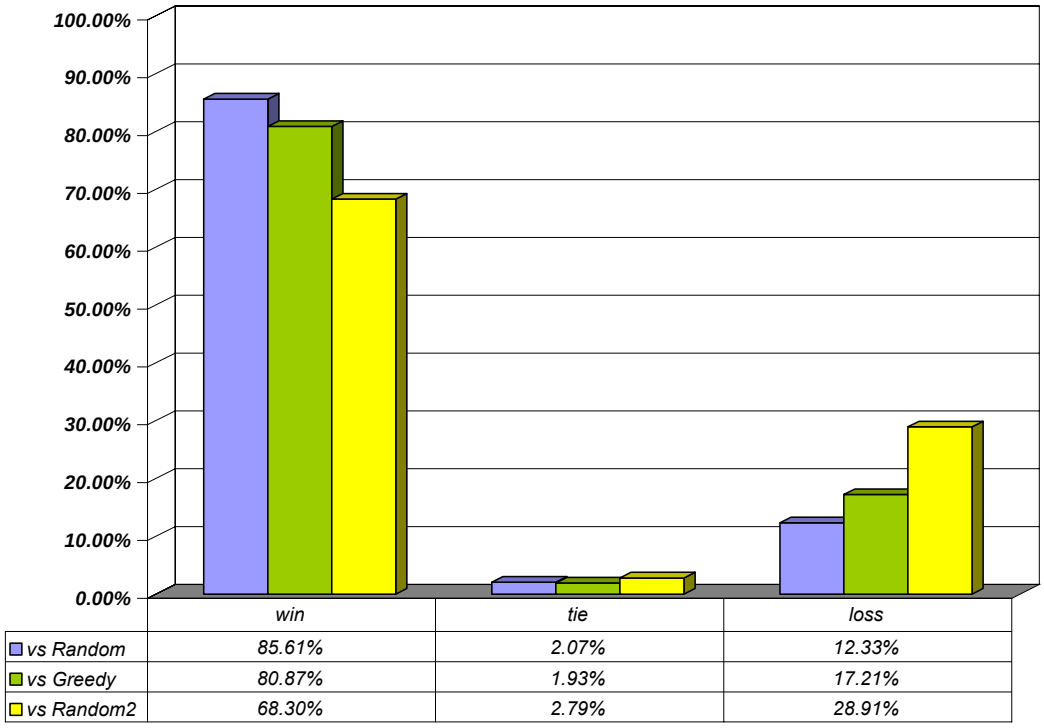


Figure 7.11 MLP using Simple against in-built opponents (1 trial).

7.3 Input comparison

Cranney reports huge discrepancies between networks playing black and those playing white, as those playing black were considerably better (Cranney 2002). Since the networks learned via self-play there should not be such a major difference. It is hard to pinpoint the exact cause of this discrepancy, but one possibility could be problems with the input representation. Indeed on further analysis, we can see that Cranney uses the Simple input representation, but without the extra input that tells whose go it is. This information is crucial to the network's analysis of the board as was demonstrated by Figure 5.6 Right in section 5.4. Originally, the Walker input used only the first 128 inputs, as it was believed that the networks will be able to deduce the extra information from the last 64 inputs. However, this assumption turned out to be incorrect, as the addition of the 129^{th} input improved the network's performance by an average of 1.6% for $\alpha = 0.01$ and up to 10% for $\alpha = 0.1$.

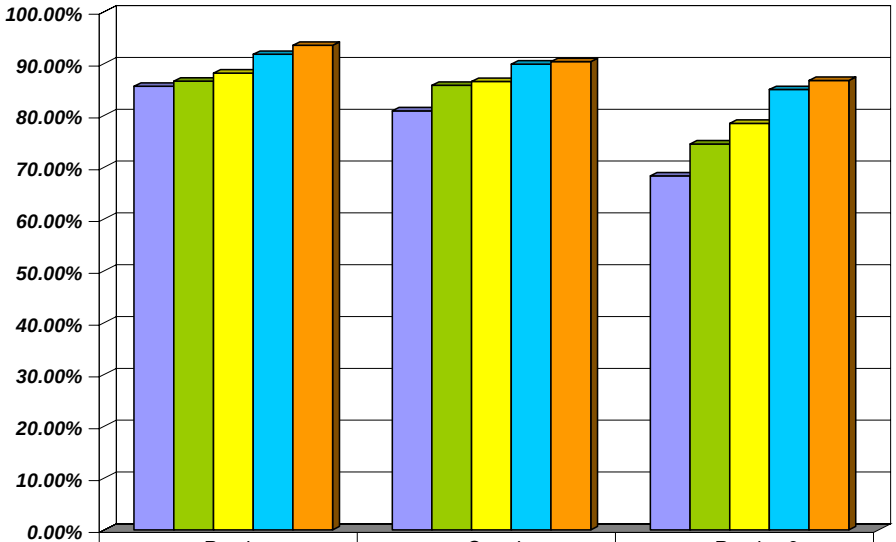
Figure 7.12 shows the percentage of wins of MLPs with various inputs playing against in-built opponents. The results are arranged in monotonic order from worst to best performance: Simple, Partial, Simple2, Walker and Advanced. Figure 7.13 shows the average winning margin for all input schemes.

Based on the winning percentage Simple2 consistently outperforms Simple. Simple and Simple2 are identical, except Simple2 uses two binary inputs per square, instead of one "ternary" input per square. This result indicates that binary inputs are easier for the network to learn, despite the fact that there are more of them. From Figure 7.13 we notice that Simple2 is worse than Simple when playing against Random and Greedy, in fact it has the worst winning margin against Greedy. It seems that Simple2 has "traded" its winning margin for better winning percentage. As suggested by Horton, the Walker and Advanced input schemes could also benefit from using binary inputs (Horton October 2005, personal communication).

The Partial scheme was specifically designed for Othello, as each row, column and diagonal of length 3 or more was connected to a separate hidden node. For some reason, this scheme performed relatively poorly, as it was only slightly better than Simple. The scheme did not manage to capture the important relationships between squares that lie on the same straight line.

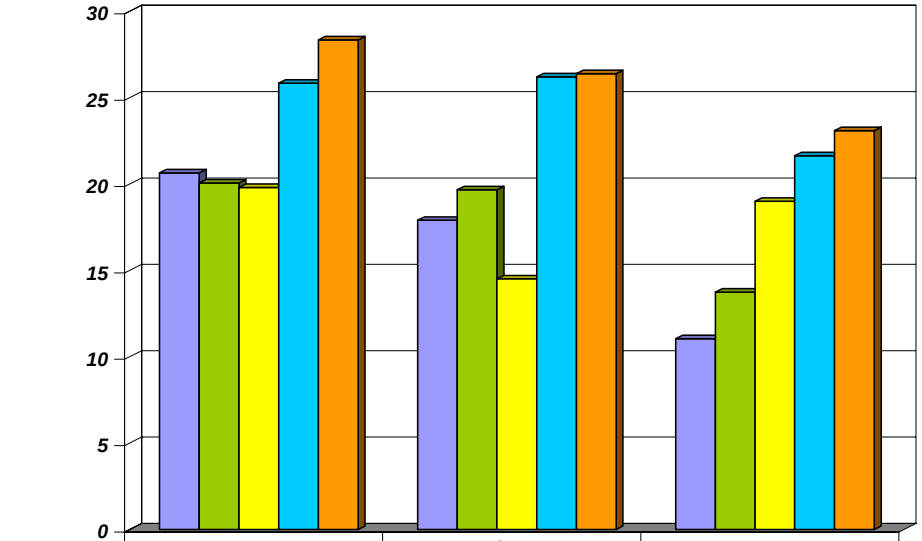
The Walker input was the best input from those that were not given any extra strategic information. The performance of Walker input is close to that of Advanced in both winning percentage and winning margin. We conclude that its superiority against simple inputs is due to the extra 64 "look-ahead" inputs.

It was no surprise that Advanced was the best input scheme. Its dominance is primarily due to the mobility information that it received. When playing against in-built opponents it often restricted their mobility, forcing them to give away corners. Once it took control of corners, it ensured that the opponent had to pass, thus maximizing its own disc count. We also notice that Walker and Advanced inputs have the strongest generalization skills as they have the least decline in performance (Figure 7.12).



	vs Random	vs Greedy	vs Random2
Simple	85.61%	80.87%	68.30%
Partial	86.59%	85.80%	74.49%
Simple2	88.15%	86.52%	78.46%
Walker	91.77%	89.88%	84.97%
Advanced	93.51%	90.36%	86.72%

Figure 7.12 MLPs with various inputs against in-built opponents (1 trial).

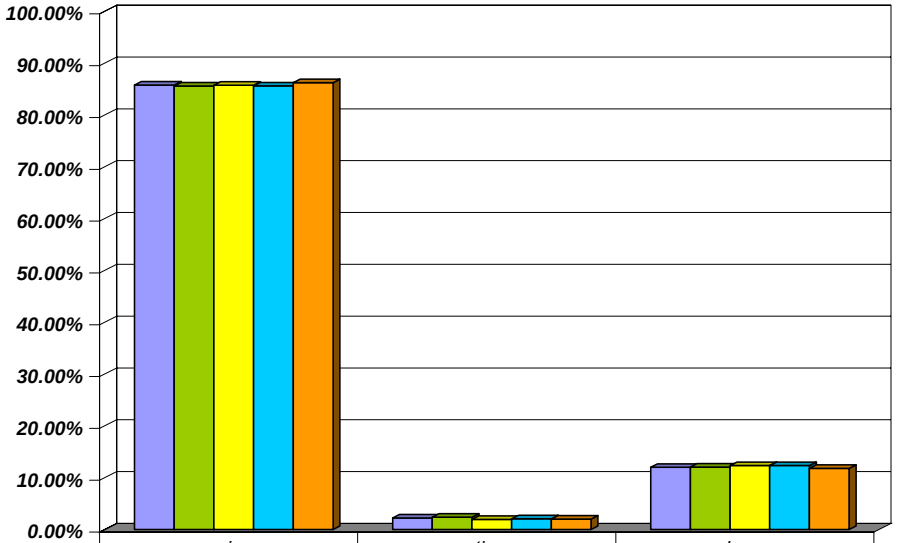


	vs Random	vs Greedy	vs Random2
Simple	20.64	17.9	11.05
Partial	20.06	19.67	13.74
Simple2	19.8	14.51	19
Walker	25.84	26.21	21.63
Advanced	28.35	26.39	23.09

Figure 7.13 Winning margin of MLPs with various inputs against in-built opponents (1 trial).

7.4 Hidden Nodes

In Figure 7.14 we compare the performance of MLPs (using Simple) with various numbers of hidden nodes: 1, 5, 8, 30 and 50. Variation between all results are less than 0.7%, which shows that the number of hidden does not make any difference. Most surprisingly, it is sufficient to have just one hidden node. This indicates that the learnt strategy is very "linear" and that there is little interaction between inputs. To examine this further, we construct a weights map for the MLP with 5 hidden nodes (Figure 7.15). The 5 hidden nodes are represented as A, B, C, D, E and the combined map is shown in F. The number in the top-right corner is the weight of that hidden node; the higher the magnitude of this weight the more that hidden node contributes to the combined weight. From Figure 7.15 we can see that hidden nodes A, C and D have not learnt anything useful as their colours are distributed randomly. Nodes B and E managed to learn the importance of corners and some edge locations. In fact, it is those nodes that contribute the most. Clearly, the network could do just as well with fewer hidden nodes, i.e. if nodes A, C, D were not used.



	win	tie	loss
1 Hidden Node	85.79%	2.22%	11.99%
5 Hidden Nodes	85.59%	2.37%	12.05%
8 Hidden Nodes	85.74%	1.94%	12.33%
30 Hidden Nodes	85.61%	2.07%	12.33%
50 Hidden Nodes	86.24%	1.99%	11.78%

Figure 7.14 MLPs using Simple with various numbers of hidden nodes playing against Random.

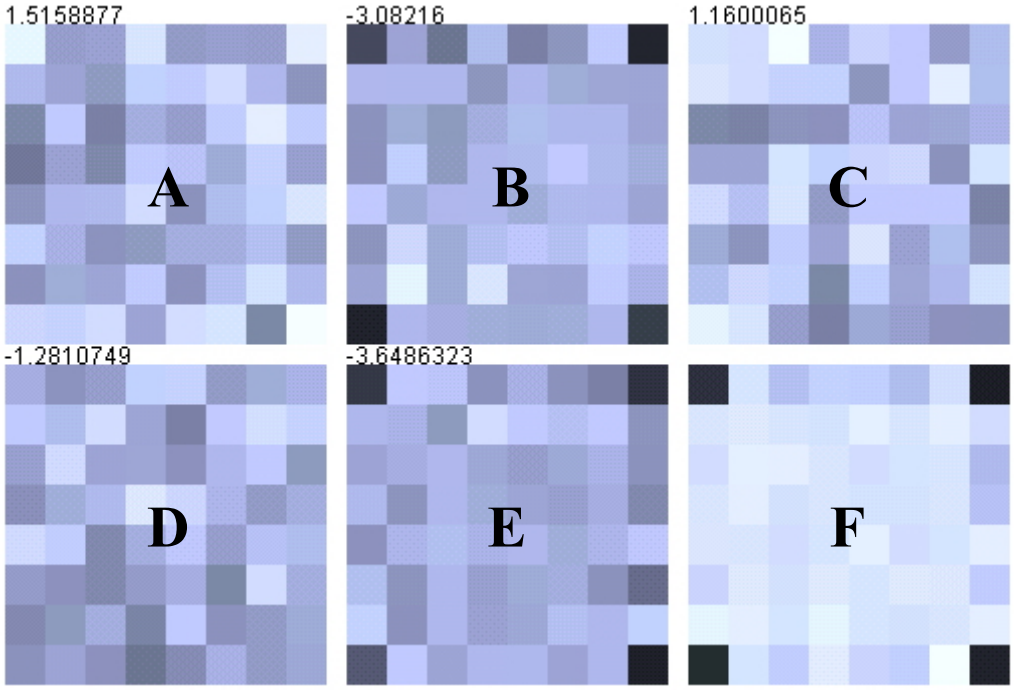
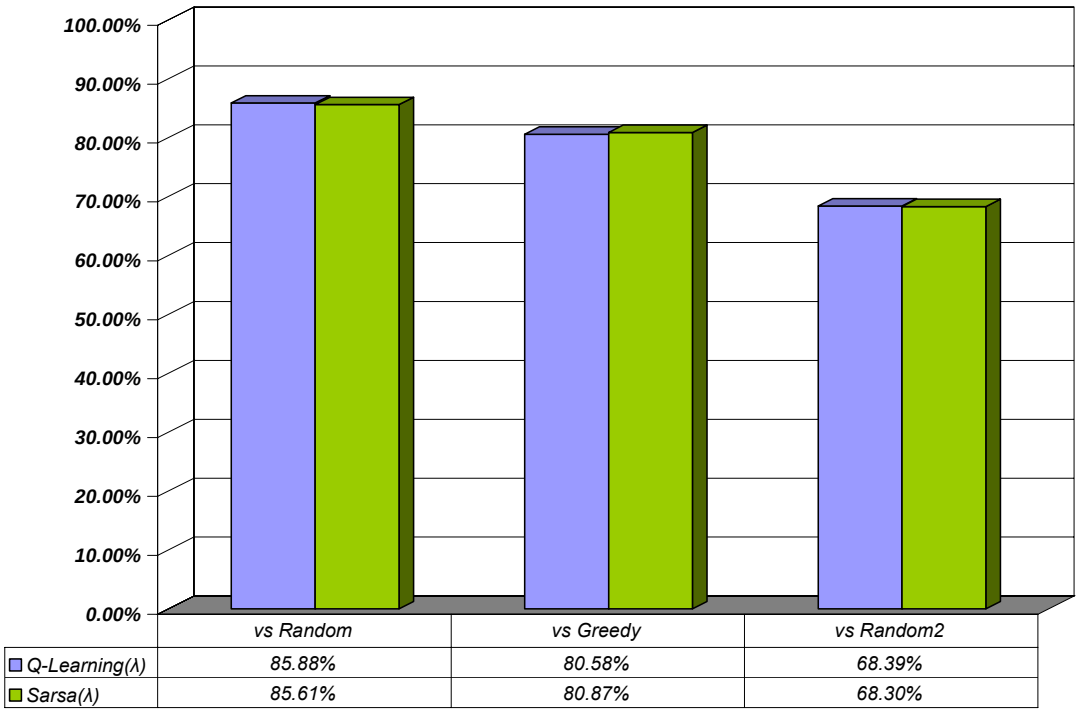


Figure 7.15 Weights map of MLP with 5 hidden nodes.

7.5 Sarsa(λ) vs Q-Learning(λ)

To compare the training algorithms and MLP using Simple input was trained with both Sarsa(λ) and Q-Learning(λ). The results are shown in Figure 7.16. Clearly the training algorithm did not make any impact on the final performance. A more noticeable difference is observed when the training algorithms are compared based on their learning curve (Figure 7.17). From Figure 7.17 we can see that Sarsa(λ) converges faster but is very unstable, while Q-Learning(λ) is slow and consistent.

Whenever an exploratory action is chosen in Q-learning(λ), the eligibility traces for all previous states are reset to zero, which should slow down the flow of information from terminating states to initial states. This is particularly true in early episodes, where we are choosing more exploratory actions. In Sarsa(λ) there is more variability in the values being learnt, as we learn from both exploratory and greedy actions. Therefore, if we have a large number of poor exploratory actions then this could reduce the quality of the policy (Dr. Vamplew October 2005, personal communication).



	vs Random	vs Greedy	vs Random2
Q-Learning(λ)	85.88%	80.58%	68.39%
Sarsa(λ)	85.61%	80.87%	68.30%

Figure 7.16 MLPs using Q-Learning and Sarsa against in-built opponents (1 trial).

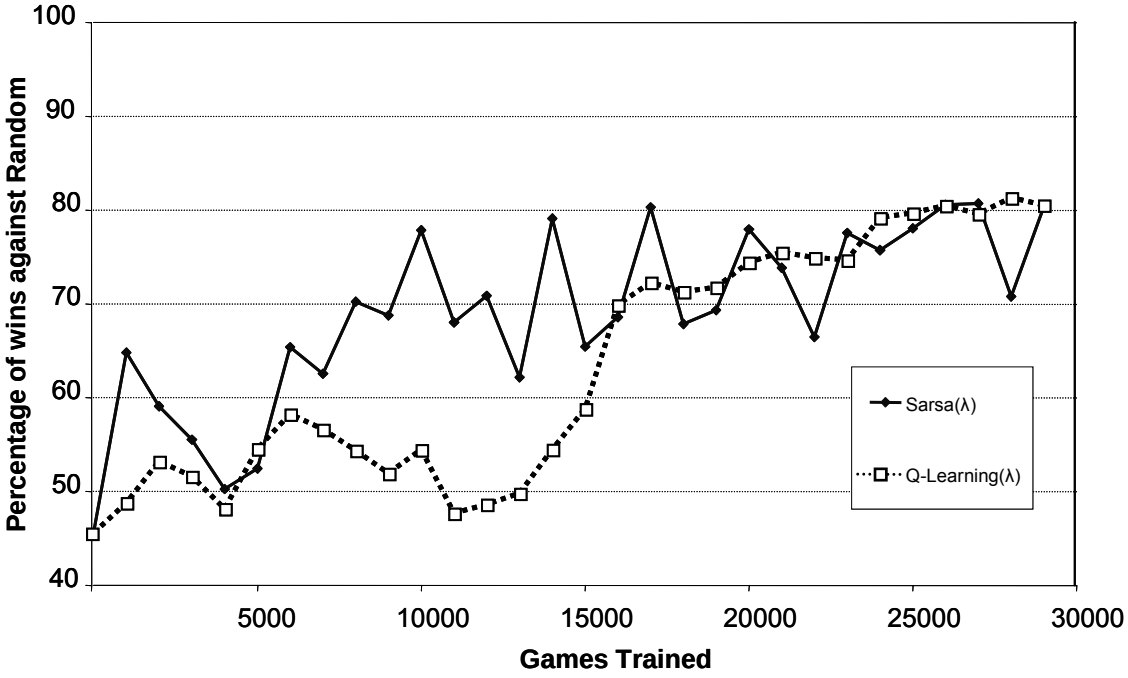


Figure 7.17 Comparison between Sarsa(λ) and Q-Learning(λ) for MLP using Simple (1 trial).

7.6 Results with RAN

There were a number of problems that caused RAN to produce inconsistent results. The first problem was that it did not add any hidden nodes. This was fixed by reducing the error threshold. The next problem with RAN was that its output was often exceeding 1 and reaching values up to 6. Reducing the learning rate did not help. On further analysis, it was found that the weights were becoming really large. An attempt was made to limit the weights by constraining them to the range [-1000, 1000]. This did not help to reduce the output. Finally, the problem was solved by making the output use a sigmoidal function. However the performance was still poor. Variation in the number of hidden nodes did not seem to make any difference.

Without doing further research it is hard to speculate why RAN did not work. One possible explanation is that RAN is not well-suited for a task that essentially uses binary inputs. It seems that the distance measure used in the RBF function performs better in tasks with continuous inputs (Dr. Vamplew October 2005, personal communication).

Chapter 8 Conclusion and Future Work

8.1 MLP vs Cascade

For slower learning rates, Cascade converges faster than MLP. The reason for this is that Cascade uses less hidden nodes during training. Its policy is not greatly affected, because the number of hidden nodes does not make any difference. Despite having faster convergence, Cascade does not actually reach better performance.

For faster learning rates, Cascade stays the same, while the MLP converges to a much better policy. The fact that Cascade works better with smaller learning rates is also supported by (Vamplew & Ollington 2005b). Since the number of weights in Cascade is increasing, it seems appropriate that its learning rate should also increase. Future work could investigate the effects of starting with $\alpha = 0.001$ and increasing it towards 0.1 every time a new node is added.

Vamplew and Ollington show that Cascade-Sarsa outperforms MLP on Acrobot and Mountain-Car, but not on the Puddleworld task (Vamplew & Ollington 2005b). A possible explanation is that in Puddleworld the puddles are convex polygons and hence require more than one hidden node to learn. Cascade-Sarsa does not meet this requirement because it adds just one node at a time. Cascade-Sarsa can be improved by training triplets of candidate nodes and then adding all three nodes to the network. Although this is likely to give better results on Puddleworld, it will probably not help much in Othello.

8.2 Learned knowledge

In summary, the networks did not learn how to:

- Play using the mobility strategy
- Predict the future outcome
- Avoid being eliminated (survival)

- Recognize corner-seizing moves

As discussed in section 5.3.3, mobility is an exceptionally difficult strategy to learn, even for human players. It requires a deep understanding of the game and is not evident from a straightforward examination. Clearly, we cannot expect the networks to learn mobility without providing them with extra information, such as the count of the available moves.

The next three items on the list are related, in that they all require a certain amount of look-ahead. Predicting the final outcome in a situation like in Figure 7.3 is not easy, because a number of major changes occur in those last moves. It is hard for networks to learn to avoid being eliminated, because for them it is simply another loss, without any features that distinguish it from other losses. Again, discounting techniques could be useful for such situations.

Given the fact that the networks learned the importance of corners and ways of protecting them, it was a surprise that they didn't learn to recognize corner-seizing moves. Similarly, it was surprising that the networks were able to eliminate the opponent, but unable to survive. Since these notions are exact opposites, a network that was trained by self-play should have either learnt both or none. In other words, this is the same as learning that 2 + 3 = 5, but not being able to see that 3 + 2 = 5. One possible explanation is that the learned value of a given board is not completely symmetric with respect to each player. Below we define the symmetry value:

$$\text{symmetry}(s) = \text{value}^{B}(s) + \text{value}^{W}(s^{-1})$$

Equation 8.1 Symmetry value of a given board s.

In Equation 8.1, $\text{value}^{B}(s)$ represents the value of the board s evaluated from black's perspective; $\text{value}^{W}(s^{-1})$ represents the value of the board s with inverted colours, evaluated from white's perspective. Now we will define the symmetry property for a game G: if G has the symmetry property then symmetry(s) = 1 for all s that occur in G. The symmetry property was tested by playing an MLP with Walker input against Random for 1,000 games. Figure 8.1 shows the average symmetry value for each stage in the game:

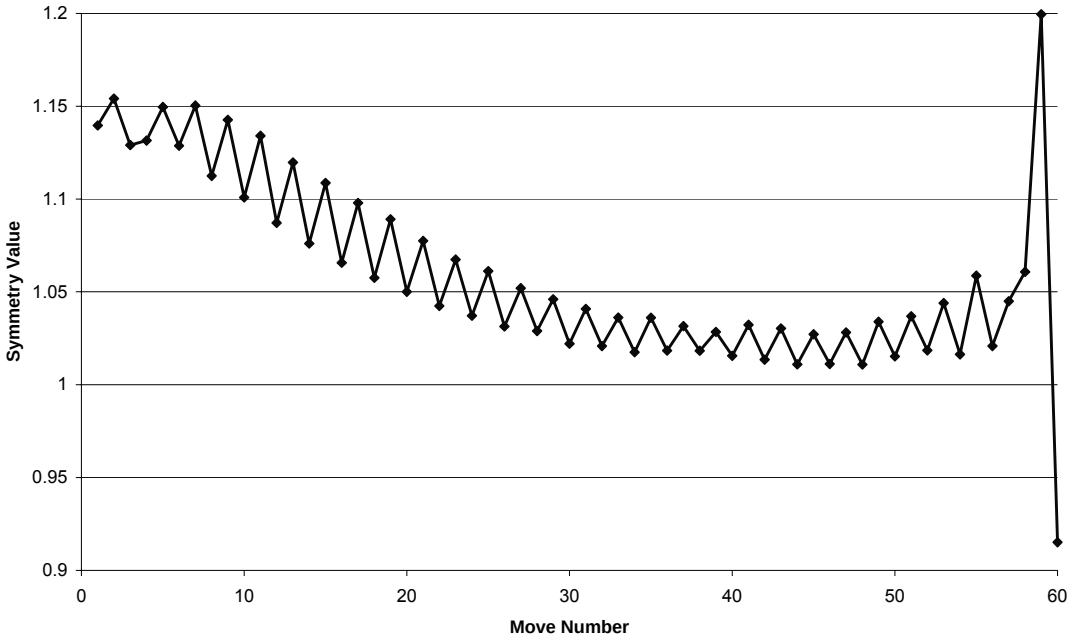


Figure 8.1 Symmetry values for each stage in the game.

From Figure 8.1 we can see that the symmetry value starts off very high and then asymptotically approaches 1, although never actually reaching it. Alarmingly, the value jumps during the last two moves. The high symmetry values for the opening moves are explained by the fact that for those moves s^{-1} is never seen by the network during training. For all other high symmetry values there is no reasonable explanation except that the network is highly biased towards one of the players.

Perhaps this bias can be eliminated if we train a separate network for each player. This way the networks will learn to evaluate positions independently, enabling the symmetry property to hold. Based on my results and those of (Deshwal & Kasera 2003), it seems that Othello is slightly biased towards the second player, and so it makes sense for each player to train separately. Training two networks in a competing environment also has disadvantages. For example, if one network becomes much stronger than the other, it will dominate game-play and as a consequence prevent the other network from learning. Ideally, we want both networks to learn at the same rate.

Alternatively, we could continue to use self-play, but evaluate states always from the perspective of one player. When the current player is black we select moves normally – those with the highest value. However, when the current player is white we evaluate

his position from black's perspective by reversing the colours on the board. This way both selections are made from black's perspective with high values corresponding to good moves.

Despite the negative results, the networks did manage to learn:

- The importance of edge locations, especially corners
- The negative outcome of going to x-squares and c-squares
- How to protect corners
- How to eliminate the opponent

The first two points show that the networks learned the positional strategy, which involves the correct evaluation of each board location. In particular the networks did really well in learning the importance of corners and ranked them 1, 2, 3 and 4 (see Figure 7.10 in section 7.2.2). However, they did not do as well for x-squares, ranking them 52, 57, 62 and 64. The bottom-left x-square (52) can cause problems, as the networks will go to that square unnecessarily often.

The last two points are of particular interest, because they show that the networks have learned more than just the positional strategy. Protecting corners and eliminating the opponent are relatively complex strategies to learn.

8.3 Input representation

It was found that Simple2 outperforms Simple. This indicates that binary inputs are easier to learn. We can expect further improvement if binary inputs are used for other inputs representations. Partial input does not perform well. Future work can involve applying partial input to Cascade. To achieve this, the Cascade networks will need to start with all the partial hidden nodes in place and add more nodes during training. Variations to the Partial input are also possible. For example, we can add hidden nodes for each block of 3x3 and 2x5 squares. This strategy was used with great success in LOGISTELLO (Buro 1994).

This study has shown that the Walker input does considerably better than Simple and nearly as well as Advanced. This indicates that the networks are benefiting from the look-ahead information provided by Walker input.

From Figure 8.2 Left, we can see that the Othello board has 8 axes of symmetry. This means that we can divide the board into 8 equivalent triangles with 10 squares each (Figure 8.2 Right). The shaded squares on the diagonals are squares that are shared between adjacent triangles. Suppose we are training the network to recognize a particular pattern on one of those triangles. Clearly, the features important to one triangle will also be important in the other 7 triangles. Leouski describes a technique called weight sharing, which utilizes this knowledge (Leouski 1995). In weight sharing, connections of the corresponding 10 squares are constrained to have the same weight. As a result the network only needs to learn 10 different weights and then share them between triangles. This technique promises to improve the accuracy of the learned policy and should be investigated in the future.

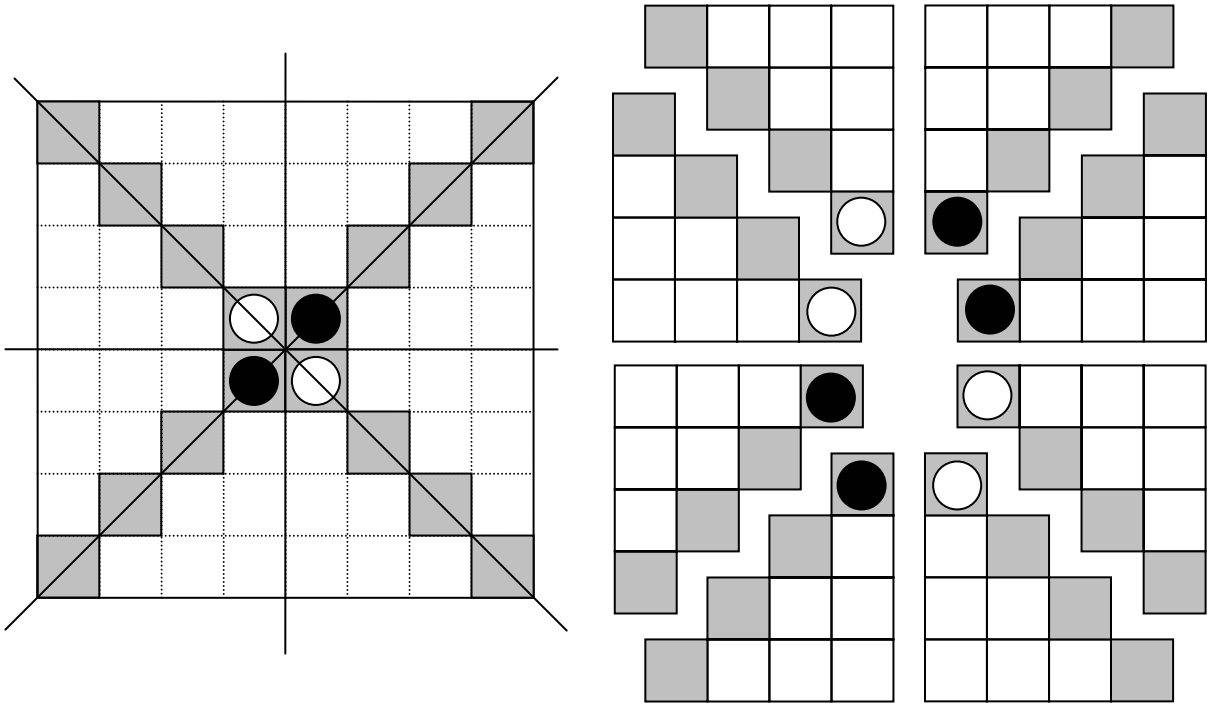


Figure 8.2 Weight sharing (Leouski 1995). Left: 8 axes of symmetry in Othello board. Right: 8 resulting triangles.

8.4 Comparison to other Othello programs

Our agent performs similar to other TD-based Othello programs. Using Simple representation our networks perform 6% better than those of Cranney. Although Cranney also uses Simple, he does not provide whose move it is, which decreases the performance of his networks.

The networks trained by Walker et al. tend to beat Windows Reversi at the Novice (2nd level), but tend to lose to Expert (3rd level). Our networks trained with Walker representation played a series of 10 games against each level of Windows Reversi (Table 8.1). Note that it was not feasible to play more games, because the source code for Windows Reversi is not publicly available. The results in Table 8.1 mirror those reported by Walker et al. Interestingly, the networks play better against Novice (2nd level) than Beginner (1st level). Perhaps the networks have learned how to beat a more advanced player like Novice, but at the same time unlearned how to beat Beginner.

	Wins	Ties	Losses
vs Beginner	6	0	4
vs Novice	8	0	2
vs Expert	2	1	7

Table 8.1 MLP using Walker against Windows Reversi.

Not surprisingly our networks perform poorly when compared to other more sophisticated programs (Deshwal & Kasera 2003; Leouski 1995; Tournavitis 2003). But this is mainly because other studies heavily rely on move look-ahead to create a stronger player. For example during the middle game, Leouski uses 2-ply search, Deshwal et al. use 4-ply search and Tournavitis uses 8-ply. Furthermore, Leouski uses exhaustive search for the last 10 moves, Deshwal et al. and Tournavitis use exhaustive search for the last 16 moves.

8.5 Number of hidden nodes

This study has shown that the number of hidden nodes in neural networks makes little difference to the performance. In fact this is one of the reasons why Cascade

converges faster than MLP. It also means that the learned strategy is rather linear. This can be improved by using a certain amount of look-ahead during training. It is also possible that 8 x 8 Othello is too complex for the networks to learn input dependencies. Future work can investigate applying these techniques to 6 x 6 Othello, as used by (Yoshioka et al. 1999).

8.6 Further improvements

In Othello there are at least three distinct stages in the game: opening, middle and end-game. Each stage lasts approximately 20 moves and requires a separate strategy. During the opening, play is generally centred on the middle squares. Stability and mobility play an important role during the middle stage, whereas the number of discs is the main factor in the end-game. We can provide extra input that describes the current stage of the game. Alternatively we can train three different networks, one for each stage, as suggested by (Leouski 1995). Furthermore, the number of game stages can be increased. For example, some authors use as many as 12 game stages (Tournavitis 2003).

Ghory suggests that an agent learns better when playing against an opponent that is stronger than itself (Ghory 2004). This way the stronger opponent can teach the agent – an approach similar to supervised learning. To achieve this, we can apply shaping (introduced in section 4.9.1). The agent can start by playing against Random. Once it has learned to beat Random, it can be trained against Greedy, Random2 and other more advanced opponents. An opponent can also be created by introducing game-tree search to the current agent (Ghory 2004).

Chapter 9 References

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